Detailed Solution

*Please note that the order of the questions differs among different test versions.*

*In the solution I used: $m_e=9.11\times10^{-31}$ kg and $h=6.626\times10^{-34}$ Js.

1. X-rays having an energy of 300 keV undergo Compton scattering from a target. The scattered rays are detected at 37 degrees relative to the incident rays. What is the energy of the scattered X-ray?

   a) 230 keV
   b) 250 keV
   c) 270 keV
   d) 290 keV
   e) 310 keV

Ans. c

\[ E = hf = \frac{hc}{\lambda}. \]

The initial wavelength is
\[ \frac{hc}{E} = \frac{6.626\times10^{-34}\times3\times10^8}{300\times10^3\times1.6\times10^{-19}} = 4.14 \times 10^{-12} \text{ m}. \]

For Compton scattering,
\[ \lambda' - \lambda_0 = \frac{h}{m_e c} (1 - \cos \theta) = \frac{6.626 \times 10^{-34}}{9.11 \times 10^{-31} \times 3 \times 10^8} (1 - \cos 37^\circ) \]
\[ = 0.49 \times 10^{-12} \text{ m} \]

\[ \therefore \text{The scattered wavelength} \]
\[ \lambda' = 4.14 \times 10^{-12} + 0.49 \times 10^{-12} = 4.63 \times 10^{-12} \text{ m} \]

The energy of the scattered X-ray
\[ E = \frac{hc}{\lambda'} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{4.63 \times 10^{-12}} = 4.29 \times 10^{-14} J = 268 \text{ keV} \]
2. Which experiment disproved the “Plum Pudding Model” of the atom?
   a) Compton Scattering
   b) Davisson-Germer Experiment
   c) Rutherford Scattering
   d) Stern-Gerlach Experiment
   e) Experiments done by Hertz establishing the photoelectric effect

   Ans. c
   Refer to the lecture note.

3. For a hydrogen atom making a transition from the n=4 state to the n=2 state, what is the wavelength of the emitted photon?

   a) 486 nm
   b) 543 nm
   c) 327 nm
   d) 284 nm
   e) 673 nm

   Ans. a
   \[ \Delta E = E_4 - E_2 = \left( -13.6 \times \frac{1}{4^2} \right) - \left( -13.6 \times \frac{1}{2^2} \right) = -13.6 \times \left( -\frac{3}{16} \right) = 2.55 \text{ eV} \]
   \[ 2.55 \text{ eV} = 4.08 \times 10^{-19} J = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{\lambda} \]
   \[ \therefore \lambda = 4.87 \times 10^{-7} m = 487 \text{ nm} \]
   *You’ll get 485~487 nm based on the digits of the constants you used.

4. An electron is confined between two impenetrable walls 0.200 nm apart. What is the minimal speed of the electron?

   a) 100 m/s
   b) 1000 m/s
   c) 10,000 m/s
   d) 100,000 m/s
   e) 1.000,000 m/s

   Ans. e
   Ground state (n=1) gives minimum energy, therefore yielding minimal speed.
   \[ E_1 = \frac{h^2}{8mL^2} = \frac{(6.626 \times 10^{-34})^2}{8 \times 9.11 \times 10^{-31} \times (0.2 \times 10^{-9})^2} = 1.5 \times 10^{-18} J \]
   \[ E_1 = \frac{1}{2}mv^2 \therefore v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \times 1.5 \times 10^{-18}}{9.11 \times 10^{-31}}} = 1.8 \times 10^6 m/s \]
   e is the closest answer.
5. If a hydrogen atom has orbital angular momentum $4.714 \times 10^{-34}$ J seconds, what is the orbital quantum number for the state of the atom?

   a) 1
   b) 2
   c) 3
   d) 4
   e) 5

   Ans. d

   $$L = \sqrt{l(l+1)} \frac{\hbar}{2\pi} = 4.714 \times 10^{-34}$$

   $\therefore \sqrt{l(l+1)} = 4.714 \times 10^{-34} \times \frac{2\pi}{6.626 \times 10^{-34}} = 4.47$

   $l(l+1) = 19.98 \approx 20$ $\therefore l = 4$

   *You will get $l(l+1) = 19.98 \sim 20.13$ based on the digits of the constants you used.

6. A photon with energy $2.28$ eV is absorbed by a hydrogen atom. What is the minimum energy level, $n$, for which the hydrogen atom can be ionized by such a photon?

   a) 1
   b) 2
   c) 3
   d) 4
   e) 5

   Ans. c

   $$E_1 = -13.6 \text{ eV}, E_2 = -3.4 \text{ eV}, E_3 = -1.51 \text{ eV}, E_4 = -0.85 \text{ eV}.$$  

   $\therefore 2.28$ eV is enough to ionize hydrogen atoms for $n \geq 3$.

7. An electron and an alpha particle both moving at nonrelativistic speeds have the same de Broglie wavelength. Which of the following are also the same for the two particles? Assume a speed of light of $3 \times 10^8$ m/s; an electron mass of $9 \times 10^{-31}$ kg; an alpha particle mass of $3.7 \text{ GeV}$, and Planck’s constant of $6.6 \times 10^{-34}$ J seconds.

   a) The speed is the same for both particles.
   b) The kinetic energy is the same for both particles.
   c) The momentum is the same for both particles.
   d) The frequency is the same for both particles.
   e) None of the above statements are correct.

   Ans. c

   $$\lambda_{\text{de Broglie}} = \frac{\hbar}{p} = \frac{\hbar}{mv} = \frac{\hbar}{\sqrt{2mE}} = \frac{\hbar}{\sqrt{2mhf}}$$

   So only the momentum ($p$) will be the same. The difference in mass makes the speed, the kinetic energy and the frequency different.
8. How many quantum states are there in the hydrogen atom for the sum of n=1,2, and 3 together?

a) 2 
b) 8 
c) 10 
d) 18 
e) 28 

Ans. e

The number of states of a quantum number n is $2n^2$

Thus the sum of the states for n=1,2,3 is $2 \times 1^2 + 2 \times 2^2 + 2 \times 3^2 = 2 + 8 + 18 = 28$

9. Molybdenum has a work function of 4.20 eV. What is the stopping potential if the incident light has a wavelength of 180 nm?

a) 2.5V 
b) 2.0V 
c) 1.5V 
d) 1.0V 
e) 0.5V 

Ans. a

$$eV_s = hf - \phi = \frac{hc}{\lambda} - \phi$$

$$1.6 \times 10^{-19} \times V_s = \frac{6.626\times 10^{-34}\times 3\times 10^8}{180\times 10^{-9}} = 4.2 \times 1.6 \times 10^{-19}, \quad \therefore V_s = 2.68 \text{ V}$$

*You’ll get $V_s = 2.68\sim2.70$ based on the digits of the constants you used.

10. According to Heisenberg’s uncertainty principle, the more accurately we know a subatomic particle’s position the less we know it’s precise:

a) energy 
b) mass 
c) quantum numbers 
d) linear momentum 
e) lifetime 

Ans. d

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$