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Accurate cross section estimates for key Standard Model processes in proton-proton collisions at $\sqrt{S} = 7$ TeV

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Abstract

This study summarizes some of the higher order Standard Model (SM) cross-sections using the latest available calculations for proton-proton collisions at 7 TeV centre-of-mass energy. The cross-section calculations are made choosing scales and parton distribution functions (PDFs) which are widely used in the CMS Collaboration for Monte Carlo simulations. The scale and PDF uncertainties are provided for these choices, along with guidelines about how to use these numbers for normalization in the analyses.

1 Introduction

The LHC has recently started delivering proton-proton collisions at a centre-of-mass energy $\sqrt{S} = 7$ TeV. Physics analyses at the LHC often depend on various inputs from theory that are only known with limited accuracy. The predictions on process cross sections are a well known example; their accuracy depends on the order of perturbation theory of the calculation, the parton distribution functions (PDFs) used, the factorization and regularization scales assumed, and the choice of Standard Model parameters.

Most often there is no unique prescription of what calculation and what input settings should be used in a given analysis when comparing to the data. This study makes use of certain conventions, suggested by the Monte Carlo simulations used in CMS, in performing the computations of higher order cross sections, along with the determination of the associated errors, and aims at establishing common reference values for most relevant SM cross sections which analyses can refer to. For reference, the term Monte Carlo in this note refers to those tools which generate complete events by adding parton showering, fragmentation, hadronization, *etc.* to simple, leading order (LO) partonic processes.

In this note, we provide a tabulation of inclusive cross section estimates for many critical Standard Model processes and a prescription for how to apply them. The note is organised as follows: in Section 2, guidelines for the calculation of K-factors based on higher order cross sections, along with given scales and PDF uncertainties, are provided. In Section 3 the assumptions made for these calculations are given, followed by the results in Section 4. Finally, in Section 5, the results are summarized.

2 Normalization factors, scale and PDFs

To be useful, our tabulation of cross sections must state the underlying "hard" process, any kinematic cuts or kinematic assumptions, the relevant set of parameters from the SM Lagrangian, the PDF set (or sets), the scale choices, and the order of the electroweak (EW) and strong (QCD) couplings in the perturbative expansion. As a general rule, the "best" (usually highest-order) available calculation should be used when calculating cross-sections along with dependencies on kinematics. These predictions may be used to normalize or reweight the Monte Carlo distributions in the analyses. In doing so, generator level cuts which may have been used for a given Monte Carlo production should be taken into account also in the calculation of the K-factors. This requires that, for instance, the reference leading order (LO) parton shower based MC and the next-to-leading order (NLO) calculation should use as much as possible the same cuts, and similar PDFs. Applying an inclusive K-factor to a (LO) Monte Carlo generation implicitly assumes that the change in acceptance introduced by the analysis is not very sensitive to higher order QCD effects. This is an aspect that should be checked carefully by those analyses where such a sensitivity may be present. Alternatives to a constant K-factor are an event reweighting, function of some kinematic variables, or a determination of an *a posteriori* K-factor, if the same acceptance cuts of the analysis can be reproduced at parton level for the higher order calculation.

Other inputs that should be made uniform between leading and higher order calculations are the normalization μ_R and factorization μ_F scales, as well as the strong coupling constant and the PDFs. Additionally, the order of the PDFs used should match with the order of the matrix-element calculations in the ratio for the K-factors.

As a specific example to illustrate the subtleties that need consideration when deciding which higher-order calculation to apply to a given lower-order process, we will focus on $pp \to W^+ + X$ production. An inclusive prediction for the total cross section is available at NNLO in QCD. This calculation could be used to normalize a leading order calculation of $pp \to W^+ + X$ in an event generator with, ideally, the same choice of EW parameters used for the LO Monte Carlo. The first consideration is whether the treatment of the W^+ boson mass is consistent with the event generator, which most likely samples from a Breit-Wigner distribution. In cases like $pp \to Z + X$, it is important to note whether the "Z" boson includes a virtual photon, and, if so, the invariant mass window used in each calculation. They will clearly have to be match, which can be easily done provided that the "higher-order" corrections refer to QCD.

Some consideration of the kinematics is also in order. At NLO, the complete calculation of $pp \to W^+X$ includes the tree-level matrix elements (MEs) for $q\bar{q}' \to W^+g$, $qg \to W^+q'$, and $\bar{q}g \to W^+\bar{q}'$, which produce a high-p_T tail in the W^+ p_T distribution. If the NLO normalization were applied to a parton shower calculation with M_W as a cutoff on the parton virtualities, the resulting differential distribution for $p_T(W^+)$ would be biased to low values. In some analyses, when one is not directly observing $p_T(W^+)$, but is focused on the ℓ^+ decay lepton, this is not a concern. Fortunately, all modern event generators include a ME-correction to the parton shower, so as to reproduce this high-p_T tail.

At NNLO, the highest-order calculation available, a complete calculation of $pp \rightarrow W^+X$ includes W^+ + 2 parton final states. Thus, the $gg \rightarrow W^+q\bar{q}'$ MEs are included. No such ME corrections are available in vanilla event generators, but would be included in a PS-ME "matched" MLM- or CKKW-like calculation (like the ones implemented in generators like Alpgen [1] or MadGraph [2] or Sherpa [3]). In practice, a "matched" calculation may contain more complicated topologies than the best inclusive higher-order calculation. The mixture of topologies will not match between the different calculations. The kinematics of q and g jets may be different, and the different radiation patterns could lead to different reconstruction efficiencies and acceptances. We view this as a subtle effect which will only become important when systematic uncertainties in the analyses become smaller than the theoretical ones.

Another consideration is the choice of PDF. Event generators are most often used with LO PDFs. The underlying event model tune, for example, may be based on a LO PDF. The same PDF is often used for determining the kinematics of the LO partonic process, the initial state radiation (ISR) parton showering, and the multiple-parton-interaction (MPI) model used to describe the underlying event activity. A standard NLO PDF would predict a very different pattern of underlying event activity, for example. Because the parton shower for $pp \rightarrow W^+X$ includes a ME correction, one may conclude that it is acceptable to use a LO PDF for this calculation with a higher-order normalization. This assumption may be wrong. Surprisingly, in many relevant cases, the primary kinematic effect of a higher-order calculation originates from the NLO PDF. The rapidity of the W^+ boson, $y(W^+)$ is modeled quite accurately with a LO ME + NLO PDF at LHC energies. On the other hand, the rapidity of the W^- or Z boson is not as sensitive.

Thus, while the tables provided are useful for guidance, some work is necessary to understand how to apply them.

2.1 Scale uncertainties

The calculation of cross-sections in a given order in perturbation theory implies a dependence on both renormalization (μ_R) and factorization (μ_F) scales. These are typically considered to be the same as the central value (μ_0) of the scale. For estimating the scale uncertainty, the scale choices are varied in the units of μ_0 in an independent way. The uncertainty on the cross section given by the scale choices is then conventionally determined by taking the maximum variation found in a range of scales $1/2\mu_0 < \mu_R < 2\mu_0; 1/2\mu_0 < \mu_F < 2\mu_0$.

It is unclear if this choice of variation is conservative or liberal, and it is to some extent arbitrary. Other prescriptions have been suggested in literature [4] but have so far been applied only to inclusive jet production, with results not very different from the conventional ones.

2.2 PDFs

In general the most recent PDF sets should be used for cross section and acceptance calculations. If an analysis acceptance is studied using PYTHIA [5] or HERWIG [6], the LO PDF (CTEQ6M [7] used in CMS simulations) should be used as a central value. However, the uncertainties on cross sections, and hence the uncertainties on acceptance, are computed with respect to the nominal choice at higher orders. We compute the PDF uncertainties using the prescription provided by the CTEQ Collaboration [7]. Additionally, the final systematics are computed using the envelopes provided by the central values and PDF errors from the MSTW08 [8], CTEQ6.6 [9] and NNPDF2.0 [10] PDFs, using each group's prescriptions for combining the two types of errors.We use the PDF4LHC working group prescriptions [11] to combine these uncertainties. In Appendix B we present this procedure in more detail, along with the method we use to also incorporate α_S uncertainties in our final error estimates.

3 Central values and choices for generator parameters

In this paper we report the results of NLO and NNLO computations of some key Standard Model processes. The NNLO cross sections for W and Z production are determined with FEWZ [12] and crosschecked with DYNNLO [13] by using the MSTW08 [8] NNLO predictions for the PDFs. The NLO cross section for the other processes are determined with MCFM 5.8 [14] and crosschecked with MC@NLO v3.4 [15]. The NLO input PDF collection used correspond to the MSTW2008 [8], CTEQ6.6 [9] and NNPDF2.0 [10] sets.

All cross-sections are determined for proton-proton collisions at a centre-of-mass energy of 7 TeV, with the input parameters chosen to be the same used for the nominal MC simulation in CMS.

Some of the parameter settings in accordence with the PDG [16] recommendation for performing the calculations are given in Table 1.

Input parameters	Values for Central Choice
PDF Set	CTEQ6M
W boson mass	80.398 GeV
W boson Width	2.141 GeV
Z boson mass	91.1876 GeV
Z boson Width	2.4952 GeV
t quark mass	172.5 GeV
b quark mass	4.8 GeV
c quark mass	1.27 GeV
fine-structure constant	0.007297352

Table 1: Input parameters used for obtaining the central value for various SM processes.

4 Higher order cross sections

This section tabulates the cross-sections of key SM processes using the setup and input settings described in the previous section. The error on all the cross-section presented includes the scale uncertainties, determined by varying the factorization and renormalization scales by a factor 2 and 0.5, and the uncertainties from the PDFs and the value of α_S, determined at NLO. For the NNLO calculations, this error is taken conservatively as giving the same relative uncertainty than the one at NLO. For the NLO computations MCFM [14] is used, and the PDF+α_S errors are determined by using the procedures from the MSTW2008 [8], CTEQ6.6 [9] and NNPDF2.0 [10] sets. The errors are then combined according to the PDF4LHC prescriptions [11].
Table 2 and 3 present a summary of the NNLO results obtained with FEWZ for W and Z/γ^* production, respectively. For W production, the individual charge-defined final states are reported separately and combined. For Z/γ^* production, three di-lepton mass intervals are calculated independently.

process	σ_{NNLO} (pb)	$\Delta\sigma_{\mathrm{PDF}+\alpha_s}$ (pb)	$\Delta\sigma_\mu$ (pb)	$\Delta\sigma_{\mathrm{NNLO}}$ (pb)
$\mathrm{pp} \to \mathrm{W}^- \to \ell^-\bar{\nu}$	12858	± 654	± 174	± 677
$\mathrm{pp} \to \mathrm{W}^+ \to \ell^+\bar{\nu}$	18456	± 850	± 233	± 881
$\mathrm{pp} \to \mathrm{W} \to \ell\bar{\nu}$	31314	± 1504	± 407	± 1558

Table 2: NNLO cross sections for W production into leptons. The corresponding branching ratio is already included. For the calculation the reference scale $\mu = \mu_R = \mu_F$ is taken as $m_{\mathrm{W}} = 80.398$. The error coming from the scale variation is dominated by the numerical precision of the integration, so the latter is quoted as error. In adding up the charged-conjugate channels the errors are summed linearly to conservatively take into account their large correlation.

process	σ_{NNLO} (pb)	$\Delta\sigma_{\mathrm{PDF}+\alpha_s}$ (pb)	$\Delta\sigma_\mu$ (pb)	$\Delta\sigma_{\mathrm{NNLO}}$ (pb)
$\mathrm{pp} \to \mathrm{Z}/\gamma \to \ell^-\ell^+$, $m_{\ell\ell} > 20\,\mathrm{GeV}$	4998	± 270	± 34	± 272
$\mathrm{pp} \to \mathrm{Z}/\gamma \to \ell^-\ell^+$, $m_{\ell\ell} > 50\,\mathrm{GeV}$	3048	± 128	± 34	± 132
$\mathrm{pp} \to \mathrm{Z}/\gamma \to \ell^-\ell^+$, $60\,\mathrm{GeV} < m_{\ell\ell} < 120\,\mathrm{GeV}$	2916	± 122	± 34	± 127

Table 3: NNLO cross sections for Z/γ production into charged leptons. The corresponding branching ratio is already included. The table presents three different cross-sections, determined in different lepton pair mass ranges. For the various calculations the reference scales $\mu = \mu_R = \mu_F$ are taken as $m_{\mathrm{Z}} = 91.1876$. The error coming from the scale variation is dominated by the numerical precision of the integration, so the latter is quoted as error.

Table 4 presents the summary of the NLO results obtained for top-pair and single top production with MCFM [17, 18]. For single top, where relevant, charged-defined states are reported separately and then combined.

The cross sections for $t\bar{t}$ are in very good agreement with $\sigma_{t\bar{t}}^{NNLL} = 165 \pm 10$ pb using NNLL resummations [20], with a top mass of 173 GeV. Similarly, the single top production in s-channel agrees well with the NNLL approximations studies [21] within uncertainties $\sigma_{Singletop}^{NNLL} = 4.6 \pm 0.06 \pm 0.13$ pb.

In table 5 we report the summary of NLO cross sections for other key SM process: vector boson production in

process	$\sigma_{\rm NLO}$ (pb)	$\Delta\sigma_{\rm PDF+\alpha_s}$ (pb)	$\Delta\sigma_\mu$ (pb)	$\Delta\sigma_{\rm NLO}$ (pb)
$\rm pp \to t\bar{t}$	157.5	$\pm$ 14.7	$^{+18.0}_{-19.5}$	$^{+23.2}_{-24.4}$
$\rm pp \to t$, s-channel	2.72	$\pm$ 0.08	$^{+0.07}_{-0.06}$	$^{+0.11}_{-0.10}$
$\rm pp \to \bar{t}$, s-channel	1.49	$\pm$ 0.08	$^{+0.04}_{-0.03}$	$^{+0.09}_{-0.08}$
$\rm pp \to t$+c.c., s-channel	4.21	$\pm$ 0.16	$^{+0.11}_{-0.09}$	$^{+0.19}_{-0.18}$
$\rm pp \to t$, t-channel	42.6	$\pm$ 2.2	$^{+0.9}_{-0.8}$	$^{+2.4}_{-2.3}$
$\rm pp \to \bar{t}$, t-channel	22.0	$\pm$ 0.8	$^{+0.6}_{-0.3}$	$^{+0.10}_{-0.8}$
$\rm pp \to t$+c.c., t-channel	64.6	$\pm$ 3.0	$^{+1.5}_{-1.1}$	$^{+3.4}_{-3.2}$
$\rm pp \to t^\pm W^\mp\ (W^\mp \to \ell^\mp \nu)$	4.66	$\pm$ 0.58	$^{+0.20}_{-0.55}$	$^{+0.61}_{-0.80}$

Table 4: NLO cross sections for inclusive top-pair and single top production. For all the calculations the reference scale $\mu = \mu_R = \mu_F$ is taken as $m_{\rm t} = 172.5$. The top quark decays inclusively. For Wt, the W only decays to lepton and the subtraction scheme of the top-pair component which is described in [19] is used. In adding up charged-conjugate channels the errors are summed linearly to conservatively take into account their large correlation.

association with heavy flavours and di-bosons. Where relevant, charged-defined states are reported separately and then combined.

process	$\sigma_{\rm NLO}$ (pb)	$\Delta\sigma_{\rm PDF+\alpha_s}$ (pb)	$\Delta\sigma_\mu$ (pb)	$\Delta\sigma_{\rm NLO}$ (pb)
$\rm pp \to W^+\bar{c} \to \ell^+\bar{\nu}\bar{c}$	-	$\pm$ 0	$^{+0.}_{-0.}$	$^{+0.}_{-0.}$
$\rm pp \to W^-c \to \ell^-\nu c$	-	$\pm$ 0	$^{+0.}_{-0.}$	$^{+0.}_{-0.}$
$\rm pp \to Wc \to \ell\nu c$	-	$\pm$ 0	$^{+0.}_{-0.}$	$^{+0.}_{-0.}$
$\rm pp \to W^+b\bar{b} \to \ell^+\bar{\nu}b\bar{b}$	-	$\pm$ 0	$^{+0.}_{-0.}$	$^{+0.}_{-0.}$
$\rm pp \to W^-b\bar{b} \to \ell^-\nu b\bar{b}$	-	$\pm$ 0	$^{+0.}_{-0.}$	$^{+0.}_{-0.}$
$\rm pp \to Wb\bar{b} \to \ell\nu b\bar{b}$	-	$\pm$ 0	$^{+0.}_{-0.}$	$^{+0.}_{-0.}$
$\rm pp \to Z/\gamma b\bar{b} \to \ell^-\ell^+b\bar{b}$, $m_{\ell\ell} > 20\,\rm GeV$	-	$\pm$ 0	$\pm$ 0	$\pm$ 0
$\rm pp \to W^+W^- \to \ell^+\bar{\nu}\ell^-\nu$	-	$\pm$ 0	$\pm$ 0	$\pm$ 0
$\rm pp \to W^+Z/\gamma \to \ell^+\bar{\nu}\ell^-\ell^+$, $m_{\ell\ell} > 40\,\rm GeV$	-	$\pm$ 0	$\pm$ 0	$\pm$ 0
$\rm pp \to W^-Z/\gamma \to \ell^-\nu\ell^-\ell^+$, $m_{\ell\ell} > 40\,\rm GeV$	-	$\pm$ 0	$\pm$ 0	$\pm$ 0
$\rm pp \to WZ/\gamma \to \ell\nu\ell^-\ell^+$, $m_{\ell\ell} > 40\,\rm GeV$	-	$\pm$ 0	$\pm$ 0	$\pm$ 0
$\rm pp \to Z/\gamma Z/\gamma \to \ell^-\ell^+\ell^-\ell^+, m_{\ell\ell} > 40\,\rm GeV$	-	$\pm$ 0	$\pm$ 0	$\pm$ 0

Table 5: LO and NLO cross sections for other key SM processes. The cross sections are computed for decays into charged leptons.

A more detailed breakdown of the cross-section presented in this section and their associated errors is presented in appendix A. Appendix B describes the details of the combination and the propagation of the error due to PDFs according to the [rescriptions given by the PDF authors and in [11]. The results presented here can serve to compute K-factors, which can be defined as the ratio N^{k}LO/LO for the analysis, keeping into accounts the comments in section 2.1.

5 Summary and conclusions

This note provides some calculations of higher-order cross-sections for several SM processes in pp collisions at 7 TeV. The cross-sections are computed using the FEWZ and MCFM calculators as a reference for a given choice of input parameters and a consistent choice of PDF. Guidelines on how these numbers should be used in the analyses to calculate K-factors have also been given. The dominant systematic uncertainties on the cross-sections are due to the uncertainties in the PDFs, which are typically of the order of $(y1-y2)\%$. The scale uncertainties within the variation of $1/2\mu_0 < \mu_R, \mu_F < 2\mu_0$ are found to be at $(x1-x2)\%$ level.

Appendix A: detailed cross section results

In this appendix we report a more detailed breakdown of the cross sections which have been presented in the text. In particular, we explicitly present all NLO cross-sections determined with the different PDF sets...

Top-Pair

$pp \to t\bar{t}$						
$\mu_R = \mu_F$	PDF	σ	$\Delta\sigma_{\mathrm{PDF}}$	$\Delta\sigma_{\alpha_s}$	$\Delta\sigma_{\mathrm{PDF}+\alpha_s}$	$\Delta\sigma_\mu$
$\mu_0 = m_t = 172.5\,\mathrm{GeV}$	MSTW2008	162.1	$^{+4.2}_{-5.4}$		$^{+6.9}_{-5.8}$	
	CTEQ6.6	150.6	± 6.4	± 4.4	± 7.8	
	NNPDF2.0	163.2	± 6.0	$^{+6.6}_{-7.0}$	$^{+8.9}_{-9.2}$	
Total PDF+α_s		157.5	± 14.7 ($\pm 9.3\,\%$)			
$\mu_0/2$	CTEQ6.6	168.6				$+18.0\,(+11.9\,\%)$
$2 \times \mu_0$	CTEQ6.6	131.1				$-19.5\,(-12.9\,\%)$
Final Number		157.5	$+23.8\,(15.1\,\%)$ $-25.0\,(15.9\,\%)$			

Single-Top

$pp \to t$, s-channel						
$\mu_R = \mu_F$	PDF	σ	$\Delta\sigma_{\mathrm{PDF}}$	$\Delta\sigma_{\alpha_s}$	$\Delta\sigma_{\mathrm{PDF}+\alpha_s}$	$\Delta\sigma_\mu$
$\mu_0 = m_t = 172.5\,\mathrm{GeV}$	MSTW2008	2.760	$^{+0.040}_{-0.076}$		$^{+0.040}_{-0.076}$	
	CTEQ6.6	2.696	± 0.058	± 0.009	± 0.059	
	NNPDF2.0	2.726	± 0.068	$^{+0.027}_{-0.015}$	$^{+0.073}_{-0.070}$	
Total PDF+α_s		2.719	± 0.082 ($\pm 3.0\,\%$)			
$\mu_0/2$	CTEQ6.6	2.765				$+0.069\,(2.6\,\%)$
$2 \times \mu_0$	CTEQ6.6	2.639				$-0.057\,(2.1\,\%)$
Final Number		2.719	$+0.108\,(4.0\,\%)$ $-0.100\,(3.7\,\%)$			

$pp \to \bar{t}$, s-channel						
$\mu_R = \mu_F$	PDF	σ	$\Delta\sigma_{\mathrm{PDF}}$	$\Delta\sigma_{\alpha_s}$	$\Delta\sigma_{\mathrm{PDF}+\alpha_s}$	$\Delta\sigma_\mu$
$\mu_0 = m_t = 172.5\,\mathrm{GeV}$	MSTW2008	1.541	$^{+0.032}_{-0.033}$		$^{+0.032}_{-0.033}$	
	CTEQ6.6	1.442	± 0.037	± 0.004	± 0.037	
	NNPDF2.0	1.482	± 0.031	$^{+0.020}_{-0.013}$	$^{+0.037}_{-0.034}$	
Total PDF+α_s		1.489	± 0.084 ($\pm 5.6\,\%$)			
$\mu_0/2$	CTEQ6.6	1.477				$+0.035\,(2.1\,\%)$
$2 \times \mu_0$	CTEQ6.6	1.415				$-0.030\,(2.4\,\%)$
Final Number		1.489	$+0.089\,(6.0\,\%)$ $-0.091\,(6.1\,\%)$			

tW

W

Z

W+heavy flavours

Z+heavy flavours

Di-bosons

Appendix B: PDF Error Calculation

In this appendix we present the methods we use in order to compute the errors on cross-sections arising from the uncertainties on the PDFs. For this we follow the procedures from the various groups [8, 9, 10], that we briefly outline in what follows. The errors are then combined according to the PDF4LHC working group prescriptions [11], that we also briefly describe at the end. We use everywhere the $68\,\%$ confidence interval (CI) sets.

The combination procedure is adapt for the NLO calculations. Due to the lack of suitable PDF sets at NNLO, only MSTW2008 being available, we estimate the relative uncertainty due to PDFs and α_s at NNLO as being the same

$pp \to t$, t-channel						
$\mu_R = \mu_F$	PDF	σ	$\Delta\sigma_{\rm PDF}$	$\Delta\sigma_{\alpha_s}$	$\Delta\sigma_{\rm PDF+\alpha_s}$	$\Delta\sigma_\mu$
$\mu_0 = m_{\rm t} = 172.5\,{\rm GeV}$	MSTW2008	41.72	${}^{+0.27}_{-0.86}$		${}^{+0.90}_{-1.18}$	
	CTEQ6.6	41.17	$\pm\,0.54$	$\pm\,0.43$	$\pm\,0.69$	
	NNPDF2.0	43.88	$\pm\,0.32$	${}^{+0.83}_{-0.84}$	${}^{+0.89}_{-0.90}$	
Total PDF+α_s		42.63	$\pm\,2.15$ ($\pm$ 5.0%)			
$\mu_0/2$	CTEQ6.6	40.33				$-0.84\,(2.0\,\%)$
$2\,\times\,\mu_0$	CTEQ6.6	42.10				$+0.93\,(2.3\,\%)$
Final Number		42.63	$+2.35\,(5.5\,\%)$		$-2.30\,(5.4\,\%)$	

$pp \to \bar{t}$, t-channel						
$\mu_R = \mu_F$	PDF	σ	$\Delta\sigma_{\rm PDF}$	$\Delta\sigma_{\alpha_s}$	$\Delta\sigma_{\rm PDF+\alpha_s}$	$\Delta\sigma_\mu$
$\mu_0 = m_{\rm t} = 172.5\,{\rm GeV}$	MSTW2008	22.15	${}^{+0.43}_{-0.34}$		${}^{+0.59}_{-0.64}$	
	CTEQ6.6	21.27	$\pm\,0.40$	$\pm\,0.29$	$\pm\,0.50$	
	NNPDF2.0	22.16	$\pm\,0.32$	${}^{+0.60}_{-0.55}$	${}^{+0.68}_{-0.64}$	
Total PDF+α_s		22.03	$\pm\,0.81$ ($\pm\,3.7\,\%$)			
$\mu_0/2$	CTEQ6.6	20.94				$-0.33\,(1.6\,\%)$
$2\,\times\,\mu_0$	CTEQ6.6	21.90				$+0.63\,(3.0\,\%)$
Final Number		22.03	$+0.89\,(4.0\,\%)$		$-1.05\,(4.8\,\%)$	

as at NLO. This rescaled uncertainty is then added in quadrature to the scale uncertainty, which is computed fully at NNLO instead, and using the MSTW2008 NNLO PDF set.

MSWT2008 PDF

The method is explained in better detail in [8]. The MSTW PDF sets (NLO and NNLO) consists of 41 subsets, the so-called central set and $2\,\times\,20$ sets where the eigenvector values have been varied around their best fit values. These values are either varied to fall into the $90\,\%$ CI (`MSTW2008(n)nlo90cl`) or into the $68\,\%$ CI (`MSTW2008(n)nlo68cl`).

The assymmetric uncertanties $(\Delta F^{\alpha_s}_{\rm PDF})_{\pm}$ on the PDF dependent observable F (e.g.the hadronic cross-section) for a given value of α_s are then computed as follows:

$$(\Delta F^{\alpha_s}_{\rm PDF})_{+} = \sqrt{\sum_{k=1}^{n} \left\{\max\left[F^{\alpha_s}(S_k^+) - F^{\alpha_s}(S_0), F^{\alpha_s}(S_k^-) - F^{\alpha_s}(S_0), 0\right]\right\}^2} \tag{1}$$

and

$$(\Delta F^{\alpha_s}_{\rm PDF})_{-} = \sqrt{\sum_{k=1}^{n} \left\{\max\left[F^{\alpha_s}(S_0) - F^{\alpha_s}(S_k^+), F^{\alpha_s}(S_0) - F^{\alpha_s}(S_k^-), 0\right]\right\}^2}, \tag{2}$$

where $F^{\alpha_s}(S_k^{\pm})$ are the values of the observable for the variation fo the PDF eigenvector k in both directions and $F^{\alpha_s}(S_0)$ is the estimate for F using the central PDF set.

$pp \to t^{\pm}W^{\mp}$ ($W^{\mp} \to \ell^{\mp}\nu$)						
$\mu_R = \mu_F$	PDF	σ	$\Delta\sigma_{\rm PDF}$	$\Delta\sigma_{\alpha_s}$	$\Delta\sigma_{\rm PDF+\alpha_s}$	$\Delta\sigma_\mu$
$\mu_0 = m_{\rm t} = 172.5\,{\rm GeV}$	MSTW2008	772.2	${}^{+27.1}_{-32.6}$		${}^{+45.8}_{-36.0}$	
	CTEQ6.6	722.1	$\pm\,32.1$	$\pm\,26.6$	$\pm\,41.7$	
	NNPDF2.0	818.2	$\pm\,37.0$	${}^{+40.4}_{-43.8}$	${}^{+54.8}_{-57.3}$	
Total PDF+α_s		776.7	$\pm\,96.3$ ($\pm\,12.4\,\%$)			
$\mu_0/2$	CTEQ6.6	630				$-92\,(12.7\,\%)$
$2\,\times\,\mu_0$	CTEQ6.6	756				$+34\,(4.7\,\%)$
Final Number		776.7	$+137.9\,(17.7\,\%)$		$-103.0\,(13.3\,\%)$	

$pp \to W^- \to \ell^- \bar{\nu}$						
$\mu_R = \mu_F$	PDF	σ	$\Delta\sigma_{\mathrm{PDF}}$	$\Delta\sigma_{\alpha_s}$	$\Delta\sigma_{\mathrm{PDF}+\alpha_s}$	$\Delta\sigma_{\mu}$
$\mu_0 = m_{\mathrm{W}} = 80.398\,\mathrm{GeV}$	MSTW2008	4.178	${}^{+0.091}_{-0.070}$		${}^{+0.091}_{-0.079}$	
	CTEQ6.6	4.099	$\pm\,0.085$	$\pm\,0.029$	$\pm\,0.090$	
	NNPDF2.0	3.955	$\pm\,0.086$	${}^{+0.060}_{-0.053}$	${}^{+0.105}_{-0.101}$	
Total PDF+α_s		4.062	$\pm\,0.208$ $(\pm\,5.1\,\%)$			
NNLO number	MSTW2008	4.286	$\pm\,0.042$ (num. int.)			
$\mu_0/2$	MSTW2008	4.270	$\pm\,0.040$ (num. int.)			$-0.074\,(1.7\,\%)$
$2\,\times\,\mu_0$	MSTW2008	4.318	$\pm\,0.042$ (num. int.)			$+0.105\,(2.4\,\%)$
Final Number		4.286	$+0.242\,(5.6\,\%)$		$-0.230\,(5.4\,\%)$	

$pp \to W^+ \to \ell^+ \nu$						
$\mu_R = \mu_F$	PDF	σ	$\Delta\sigma_{\mathrm{PDF}}$	$\Delta\sigma_{\alpha_s}$	$\Delta\sigma_{\mathrm{PDF}+\alpha_s}$	$\Delta\sigma_{\mu}$
$\mu_0 = m_{\mathrm{W}} = 80.398\,\mathrm{GeV}$	MSTW2008	5.932	${}^{+0.128}_{-0.097}$		${}^{+0.128}_{-0.122}$	
	CTEQ6.6	6.033	$\pm\,0.113$	$\pm\,0.046$	$\pm\,0.122$	
	NNPDF2.0	5.774	$\pm\,0.145$	${}^{+0.076}_{-0.069}$	${}^{+0.164}_{-0.161}$	
Total PDF+α_s		5.884	$\pm\,0.271$ $(\pm\,4.6\,\%)$			
NNLO number	MSTW2008	6.152	$\pm\,0.054$ (num. int.)			
$\mu_0/2$	MSTW2008	6.170	$\pm\,0.059$ (num. int.)			$-0.052\,(1.0\,\%)$
$2\,\times\,\mu_0$	MSTW2008	6.157	$\pm\,0.056$ (num. int.)			$+0.073\,(1.2\,\%)$
Final Number		6.152	$+0.292\,(4.8\,\%)$		$-0.290\,(4.7\,\%)$	

To estimate the additional uncertainty arising from varying the strong coupling constant value α_s we proceed as follows. The constant α_s is varied in the band $[\alpha_s^0 - 1\sigma, [\alpha_s^0 + 1\sigma]$ around its best fit value α_s^0, and PDF sets (including the $2\,\times\,20$ error sets) are derived. In principle for each of these sets the expected value for F would need to be computed along the lines above, however it has been shown that using the extreme values $\alpha_s^0 - 1\sigma$ and $\alpha_s^0 + 1\sigma$ is a good enough approximation. In our procedure we thus repeat the above calculations for the two additional PDF sets (for each subset) `MSTW2008(n)nlo90cl_asmz+68cl` and `MSTW2008(n)nlo90cl_asmz-68cl`, resulting in an (asymmetric) error band for each choice of α_s.

The combined error can then be estimated as

$$(\Delta F_{\mathrm{PDF}+\alpha_s})_+ = \max_{\alpha_s} \left\{ F^{\alpha_s}(S_0) + (\Delta F^{\alpha_s}_{\mathrm{PDF}})_+ \right\} - F^{\alpha_s^0}(S_0) \tag{3}$$

and

$$(\Delta F_{\mathrm{PDF}+\alpha_s})_- = F^{\alpha_s^0}(S_0) - \min_{\alpha_s} \left\{ F^{\alpha_s}(S_0) - (\Delta F^{\alpha_s}_{\mathrm{PDF}})_- \right\} \tag{4}$$

The final estimate for the observable F and the associated undertainties from varying PDFs and the coupling constant α_s is:

$$F^{\alpha_s^0}(S_0)^{+\,(\Delta F_{\mathrm{PDF}+\alpha_s})_+}_{-\,(\Delta F_{\mathrm{PDF}+\alpha_s})_-}. \tag{5}$$

$pp \to W^\pm \to \ell^\pm \nu$						
$\mu_R = \mu_F$	PDF	σ	$\Delta\sigma_{\mathrm{PDF}}$	$\Delta\sigma_{\alpha_s}$	$\Delta\sigma_{\mathrm{PDF}+\alpha_s}$	$\Delta\sigma_{\mu}$
$\mu_0 = m_{\mathrm{W}} = 80.398\,\mathrm{GeV}$	MSTW2008	10.111	${}^{+0.215}_{-0.160}$		${}^{+0.215}_{-0.197}$	
	CTEQ6.6	10.127	$\pm\,0.198$	$\pm\,0.074$	$\pm\,0.211$	
	NNPDF2.0	9.725	$\pm\,0.226$		${}^{+}_{-}$	
Total PDF+α_s			$\pm$ $(\pm\,\%)$			
NNLO number	MSTW2008		$\pm$ (num. int.)			
$\mu_0/2$	MSTW2008		$\pm$ (num. int.)			$(\,\%)$
$2\,\times\,\mu_0$	MSTW2008		$\pm$ (num. int.)			$(\,\%)$
Final Number			$(\,\%)$		$(\,\%)$	

$pp \to Z/\gamma \to \ell^-\ell^+$, $m_{\ell\ell} > 20\,\mathrm{GeV}$						
$\mu_R = \mu_F$	PDF	σ	$\Delta\sigma_{\mathrm{PDF}}$	$\Delta\sigma_{\alpha_s}$	$\Delta\sigma_{\mathrm{PDF}+\alpha_s}$	$\Delta\sigma_\mu$
$\mu_0 = m_Z = 91.1876\,\mathrm{GeV}$	MSTW2008	1.711	$^{+0.034}_{-0.025}$		$^{+0.034}_{-0.039}$	
	CTEQ6.6	1.732	$\pm\,0.035$	$\pm\,0.016$	$\pm\,0.038$	
	NNPDF2.0	1.642	$\pm\,0.042$	$^{+0.029}_{-0.022}$	$^{+0.051}_{-0.048}$	
Total PDF+α_s		1.682	$\pm\,0.088$ ($\pm\,5.4\,\%$)			
NNLO number	MSTW2008	1.666	$\pm\,0.008$ (num. int.)			
$\mu_0/2$	MSTW2008		$\pm$ (num. int.)			$+\,(\,\%)$
$2 \times \mu_0$	MSTW2008		$\pm$ (num. int.)			$-\,(\,\%)$
Final Number			$+\,(\,\%)$		$-\,(\,\%)$	

$pp \to Z/\gamma \to \ell^-\ell^+$, $m_{\ell\ell} > 50\,\mathrm{GeV}$						
$\mu_R = \mu_F$	PDF	σ	$\Delta\sigma_{\mathrm{PDF}}$	$\Delta\sigma_{\alpha_s}$	$\Delta\sigma_{\mathrm{PDF}+\alpha_s}$	$\Delta\sigma_\mu$
$\mu_0 = m_Z = 91.1876\,\mathrm{GeV}$	MSTW2008	0.9879	$^{+0.0198}_{-0.0151}$		$^{+0.0198}_{-0.0186}$	
	CTEQ6.6	0.9900	$\pm\,0.0182$	$\pm\,0.0067$	$\pm\,0.0194$	
	NNPDF2.0	0.9517	$\pm\,0.0201$	$^{+0.0130}_{-0.0113}$	$^{+0.0239}_{-0.0230}$	
Total PDF+α_s		0.9691	$\pm\,0.0404$ ($\pm\,4.2\,\%$)			
NNLO number	MSTW2008	1.016	$\pm\,0.004$ (num. int.)			
$\mu_0/2$	MSTW2008		$\pm$ (num. int.)			$+\,(\,\%)$
$2 \times \mu_0$	MSTW2008		$\pm$ (num. int.)			$-\,(\,\%)$
Final Number			$+\,(\,\%)$		$-\,(\,\%)$	

CTEQ6.6 PDF

The method is explained in better detail in [9]. The CTEQ6.6 PDF set consists of 45 subsets, the so-called central set and 2×22 sets where the eigenvector values have been varied around their best fit values. These values are varied to fall into the $90\,\%$ CI (`cteq66`). This implies that we will have to rescale the error obtained with this set by a factor of 1.645 to approxiamte the band corresponding to the desired $68\,\%$ CI (see also [11]).

In the CETQ case the (symmetric) error arising from the PDFs uncertainty on any PDF dependent observable F is defined as:

$$(\Delta F_{\mathrm{PDF}})_\pm = \frac{1}{2}\sqrt{\sum_{i=1}^{d}\left(F_i^{(+)} - F_i^{(-)}\right)^2}, \tag{6}$$

where $F_i^{(\pm)}$ are the values of F when varying the eigenvector i in the $(\pm)$ direction.

To estimate the uncertainty from varying α_s, the CTEQ group provided the PDF set `cteq66alphas`, which contains 5 subsets, each corresponding to varying α_s around the mean value of $\alpha_s = 0.118$. To estimate the $68\,\%$ CL error we vary α_s within $\pm\,0.002$, i.e. in the band $[A_{-2}, A_2] := [0.116, 0.120]$. The error is then calculated as:

$$(\Delta F_{\alpha_s})_\pm = \frac{1}{2}\sqrt{\left(F_0(A_{-2}) - F_0(A_2)\right)^2}. \tag{7}$$

$pp \to Z/\gamma \to \ell^-\ell^+$, $60\,\mathrm{GeV} < m_{\ell\ell} < 120\,\mathrm{GeV}$						
$\mu_R = \mu_F$	PDF	σ	$\Delta\sigma_{\mathrm{PDF}}$	$\Delta\sigma_{\alpha_s}$	$\Delta\sigma_{\mathrm{PDF}+\alpha_s}$	$\Delta\sigma_\mu$
$\mu_0 = m_Z = 91.1876\,\mathrm{GeV}$	MSTW2008	0.9496	$^{+0.0191}_{-0.0146}$		$^{+0.0.0191}_{-0.0175}$	
	CTEQ6.6	0.9518	$\pm\,0.0174$	$\pm\,0.0065$	$\pm\,0.0186$	
	NNPDF2.0	0.9147	$\pm\,0.0191$	$^{+0.0125}_{-0.0107}$	$^{+0.0288}_{-0.0219}$	
Total PDF+α_s		0.9316	$\pm\,0.0388$ ($\pm\,4.2\,\%$)			
NNLO number	MSTW2008	971.9	$\pm\,4.6$ (num. int.)			
$\mu_0/2$	MSTW2008		$\pm$ (num. int.)			$+\,(\,\%)$
$2 \times \mu_0$	MSTW2008		$\pm$ (num. int.)			$-\,(\,\%)$
Final Number			$+\,(\,\%)$		$-\,(\,\%)$	

The total error is therefore:

$$(\Delta F^{90\,\%}_{\mathrm{PDF}+\alpha_s})_\pm = \sqrt{(\Delta F_{\mathrm{PDF}})^2_\pm + (\Delta F_{\alpha_s})^2_\pm}. \tag{8}$$

which is then rescaled by the factor 1.645 to get numbers comparable to MSTW:

$$(\Delta F_{\mathrm{PDF}+\alpha_s})_\pm = \frac{1}{1.645}\sqrt{(\Delta F_{\mathrm{PDF}})^2_\pm + (\Delta F_{\alpha_s})^2_\pm}. \tag{9}$$

NNPDF2.0

The method is explained in better detail in [10]. In the NNPDF approach the uncertainties from PDFs are computed as 100 (N_{rep}) replica PDF sub-sets in the PDF set NNPDF20_100. The estimate for the mean and error on the observable PDF dependent value is then:

$$F_0 = \frac{1}{N_{\mathrm{rep}}}\sum_{i=1}^{N_{\mathrm{rep}}} F_i, \tag{10}$$

and

$$(\Delta F_{\mathrm{PDF}})_\pm = \sqrt{\frac{1}{N_{\mathrm{rep}}}\sum_{i=1}^{N_{\mathrm{rep}}}(F_0 - F_i)^2}. \tag{11}$$

The error due to varying α_s is estimated using the dedicated sets NNPDF20_0117_100 and NNPDF20_0121_100 (each again with 100 replica sets), where $alpha_s$ has been varied in the interval $[0.117, 0.121]$:

$$(\Delta F_{\alpha_s})_\pm = F^{\alpha_s \pm 0.002} - F^{\alpha_s = 0.119}. \tag{12}$$

The final error is then determined as a quadrature sum:

$$(\Delta F_{\mathrm{PDF}+\alpha_s})_\pm = \sqrt{(\Delta F_{\mathrm{PDF}})^2_\pm + (\Delta F_{\alpha_s})^2_\pm}. \tag{13}$$

Combining the 3 PDF sets

We are following the prescription by the PDF4LHC working group [11] to combine the results for the three default PDF choices. The final error reads:

$$F_{\mathrm{tot}} = \frac{1}{2}\left(\max_i\left(F^i + \left(\Delta F^i_{\mathrm{PDF}+\alpha_s}\right)_+\right) + \min_i\left(F^i - \left(\Delta F^i_{\mathrm{PDF}+\alpha_s}\right)_-\right)\right), \tag{14}$$

and

$$\Delta F_{\mathrm{tot}} = \frac{1}{2}\left(\max_i\left(F^i + \left(\Delta F^i_{\mathrm{PDF}+\alpha_s}\right)_+\right) - \min_i\left(F^i - \left(\Delta F^i_{\mathrm{PDF}+\alpha_s}\right)_-\right)\right), \tag{15}$$

where i runs over the three PDF sets MSTW, CTEQ and NNPDF.

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