

Physics 2c

Lecture 8

# ***Entropy***

***Quality of Energy***

***Free expansion of a gas***

***Microscopic Interpretation of Entropy***

# Aside:

- Today's discussion of the microscopic perspective on Entropy is not covered in our textbook. It is covered in a number of other, often somewhat more advanced textbooks. E.g. Young & Freedman, University Physics, 11th Edition, chapter 20.8.

# Order and Information

- Your dorm room is in perfect order when you know where everything is without having to look. (Perfect order can exist in a very messy room!)
- As the room gets less orderly you lose information of where things are, you start having to search for things.

Today's topic

***Why does your dorm room strive towards decreasing order?***

# Entropy

- Entropy is a quantitative measure of “order” in a system.

**More Entropy = less order**

- It's basic concept can be applied in multiple ways across different disciplines.
  - Interesting example: Information theory and “Shannon Entropy” especially in the context of digital compression algorithms in computer science.
- We'll restrict ourselves to introducing the concept and discussing it within the context of thermodynamics.

# Quality of Energy: Mixing of water

Take 1kg of water at 100 degree Celsius,  
and toss it into

1kg of water at 0 degree Celsius.

You get 2kg of water at 50 degrees Celsius.

You got the same energy but the initial system of two baths could have been used to do work.

Whereas a system of two baths at the same temperature can NOT be used as a heat engine.

# Defining Entropy difference

Assume your system changes from state 1 to state 2.  
The Difference in Entropy is defined as:

$$\Delta S = \int_1^2 \left( \frac{dQ}{T} \right)$$

Where the **integral** may go **via any reversible process**.  
Entropy is thus a characteristic of the state, and entropy difference is **independent of how we go from 1 to 2 !!!**

Let's start with a fluid.

Example: Mixing of water at different T.

Suppose 1kg of water at 100<sup>0</sup>C is placed in thermal contact with 1kg of water at 0<sup>0</sup>C.

What is the total change in entropy?

$$(c_w=4190\text{J}/(\text{kg}\cdot\text{K}))$$

$$dQ = m \cdot c_w \cdot dT$$

$$\frac{dQ}{T} = m \cdot c_w \frac{dT}{T}$$

$$\Delta S = \Delta S_{hot} + \Delta S_{cold}$$

$$\Delta S = 1\text{kg} \cdot 4190 \frac{\text{J}}{\text{kg} \cdot \text{K}} \left( \int_{373\text{K}}^{323\text{K}} \frac{dT}{T} + \int_{273\text{K}}^{323\text{K}} \frac{dT}{T} \right)$$

Hot cooling down

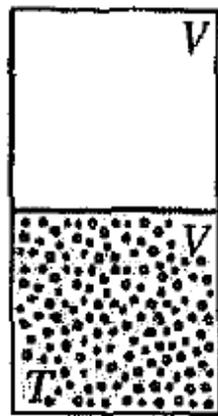
Cold heating up

$$\Delta S = -603\text{J}/\text{K} + 705\text{J}/\text{K} = 102\text{J}/\text{K}$$

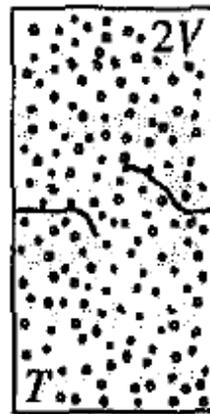
# As expected

- S increases (by 102 J/K)
- “order” decreases because the “fast” (i.e. hot) water molecules can now no longer be separated from the “slow” (i.e. cold) ones. The mixed up water is all “one big mess”.

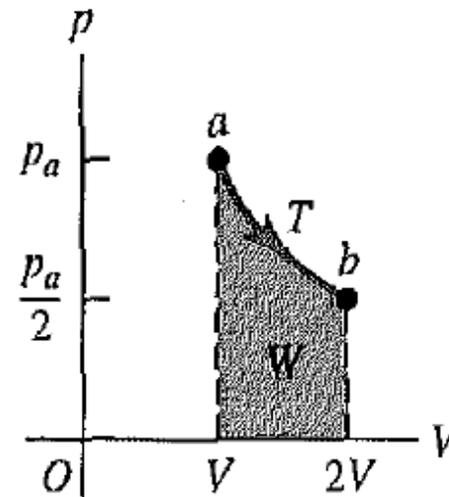
# Entropy change in free expansion



(a)



(b)



(c)

**20.15** (a,b) Free expansion of an insulated ideal gas. (c) The free-expansion process doesn't pass through equilibrium states from  $a$  to  $b$ . However, the entropy change  $S_b - S_a$  can be calculated by using the isothermal path shown or *any* reversible path from  $a$  to  $b$ .

Calculate  $\Delta S$  assuming  
isothermal expansion

$$Q = W = nRT \ln \frac{2V}{V}$$

$$Q = nRT \ln 2$$

$$\Delta S = \frac{1}{T} \int dQ = \frac{Q}{T} = nR \ln 2$$

# Macroscopic vs Microscopic

- We calculated  $\Delta S$  based on macroscopic view:

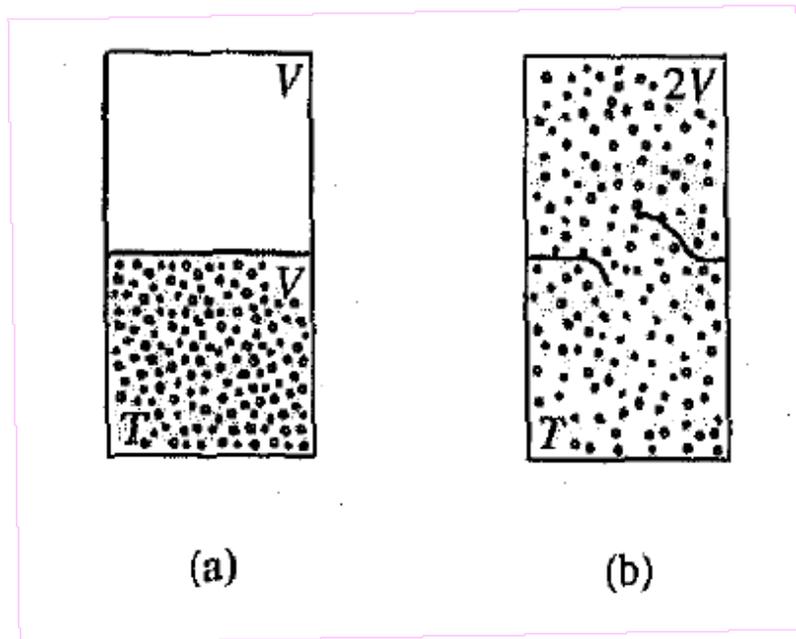
$$\Delta S = \int \frac{dQ}{T}$$

- Next, we do the same based on microscopic view.

# Microscopic definition of entropy

- $S = k \ln w$
- Where  $w$  is the number of microscopic states that share the same macroscopic state.
- Note: Macroscopically, we define only entropy differences, while microscopically we actually define the absolute value of  $S$ , rather than only  $\Delta S$ .

# Entropy change in free expansion

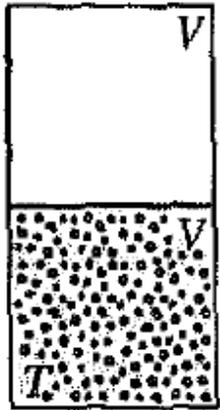


We can now understand the entropy difference in terms of the difference in the number of microscopic states shared by the same macroscopic state.

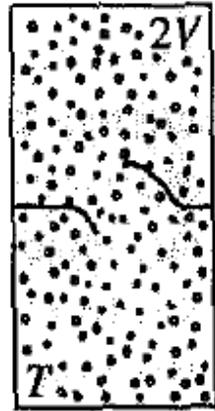
$$S = k \ln w$$

*$W = \#$  of microscopic states that share the same macroscopic state.*

# Calculating the odds



(a)



(b)

Each molecule may be on either side of the divide.  
Each molecule has twice as many Microscopic states if the volume is doubled.

$$w_2 = 2^N w_1$$

# Free expansion of ideal gas.

$$\Delta S = S_2 - S_1 = k \ln \frac{w_2}{w_1}$$

$$w_2 = 2^N w_1$$

$$\Delta S = k \cdot N \cdot \ln 2 = nR \ln 2$$

# Macroscopic vs Microscopic

- We have thus shown that the two definitions of entropy lead to the same result for the entropy change in a free expansion of an ideal gas.
- I hope the microscopic view has provided you with an intuitive understanding of what entropy means, while the macroscopic is the way we generally calculate entropy changes.

# Some things you can do with this:

- Calculate the free expansion from  $V \rightarrow 3V$ .
- Calculate the entropy change of the universe because your house is losing energy in the winter at some fixed rate.
- Calculate the entropy change due to freezing, melting, etc. etc.
- ..... and many more ...

# Entropy in a cyclic process

**Entropy = measure of likelihood of a macroscopic state.**

**Reversible cyclic process by definition returns to the same macroscopic state.**

**=> Entropy of the system is the same at the beginning and end of a cyclical reversible macroscopic process.**

# Entropy & 2<sup>nd</sup> law of thermo

No process is possible in which the total entropy decreases, after all systems taking part in the process are included.

*The entropy of the universe can only increase!*

*Macroscopically, the universe changes towards more likely outcomes.*

Why does my bedroom strive  
towards the disorderly state?

*Because there are many more ways of  
being disorderly than orderly!*

# Not all energy is created equal

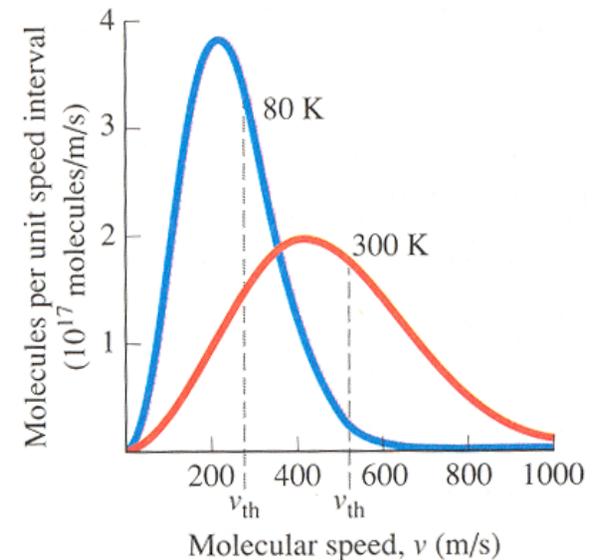
- If you give me energy, I want it in a form such that it is useful for doing work.
  - This means I want it in form of a hot and cold bath, with *maximum*  $dT$  between the two.
- Let's look at our model of an ideal gas to see what this means.

# Ideal Gas Recap

- $\langle E_{\text{kin}} \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT$
- Observation: T is positive definite !!!
- Distribution of velocities:

*Ideally, I'd like my Gas nicely ordered,  
in buckets with a small  $v^2$  range.*

*Unfortunately, this is  
fundamentally not possible.*



# Maxwell's daemon

- If this strikes your fancy, consider reading up on it in Wikipedia:
- [http://en.wikipedia.org/wiki/Maxwell's\\_demon](http://en.wikipedia.org/wiki/Maxwell's_demon)
- It's philosophically fascinating, but in the end, the second law of thermo wins.