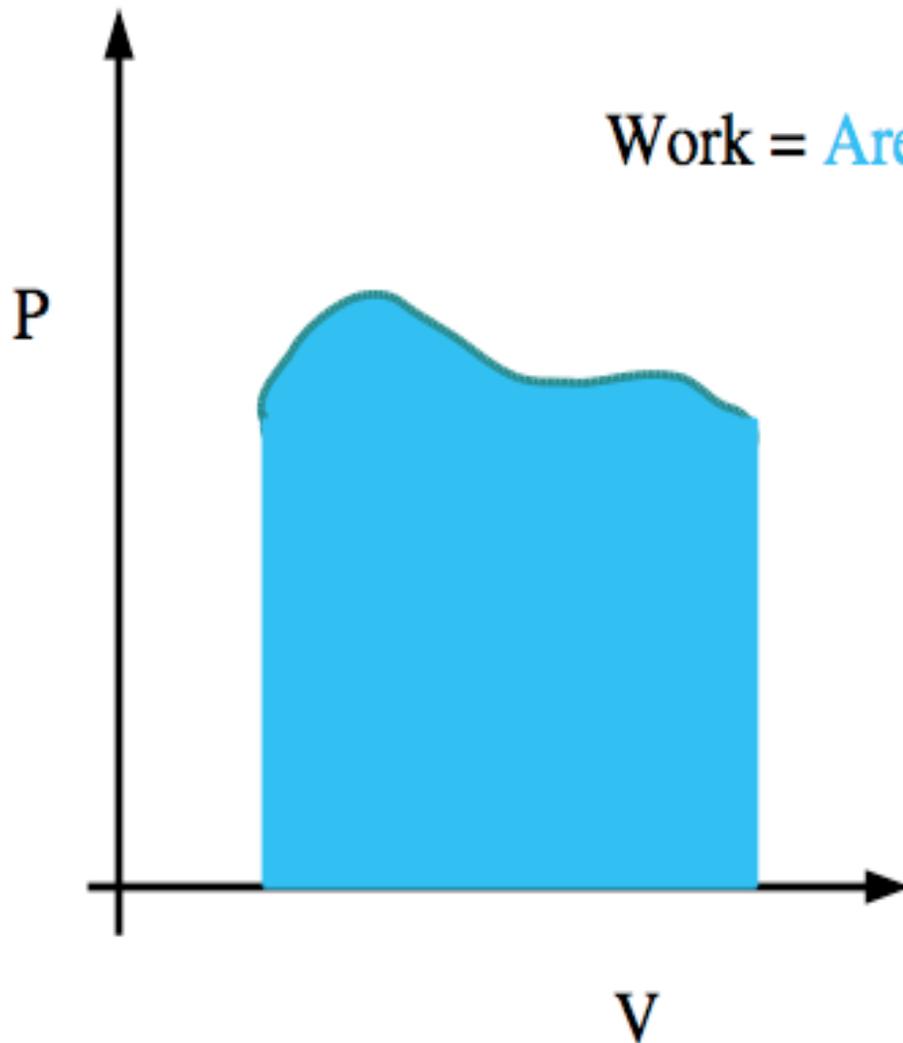


# Physics 2c Lecture 5

## Chapter 21

Thermodynamic processes  
Specific Heat of an ideal gas  
Equipartition Theorem

# PV diagram and Work



Work = Area below the curve

*Gas does work if it expands.*

$$W > 0$$

*Work is done on gas if it shrinks.*

$$W < 0$$

# Work in cyclic processes

- You have two identical cycles in the PV diagram.
- One is traversed clockwise, the other counter-clockwise.
- For which of the two does the gas do more work?
  - (a) The clockwise one
  - (b) The counterclockwise one
  - (c) Neither, because in both cases the work done by the gas is the same.

# Favorite 4 types of processes

- Isothermal:  $T=\text{const}$        $dU=0$
- Isobaric:       $P=\text{const}$
- Isochore:       $V=\text{const}$        $W=0$
- Adiabatic:       $dQ=0$

***This covers all the players in the game!***

# 4 Processes

**TABLE 21-1** Ideal Gas Processes

	ISOTHERMAL	CONSTANT-VOLUME	ISOBARIC	ADIABATIC
PV diagram				
Defining characteristic	$T = \text{constant}$	$V = \text{constant}$	$P = \text{constant}$	$Q = 0$
First law	$Q = W$	$Q = \Delta U$	$Q = \Delta U + W$	$\Delta U = -W$
Work done by gas	$W = nRT \ln\left(\frac{V_2}{V_1}\right)$	0	$W = P(V_2 - V_1)$	$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$
Other relations	$PV = \text{constant}$	$Q = nC_V \Delta T$	$Q = nC_p \Delta T$ $C_p = C_V + R$	$PV^\gamma = \text{constant}$ $TV^{\gamma-1} = \text{constant}$

Isothermal ( $dU=0$ )

$$P = \frac{nRT}{V}$$

$$Q = W = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln \frac{V_f}{V_i}$$

## Isochoric ( $dV=0$ )

- Constant volume  $\Rightarrow W=0$
- $\Rightarrow Q=dU$
- Define “**Molar specific heat at constant volume**”:

$$C_v = \frac{Q}{n\Delta T} = \frac{1}{n} \frac{\Delta U}{\Delta T}$$

# Specific Heat for Ideal Monoatomic Gas

- $U = N \langle E_{\text{kin}} \rangle$
  - From ideal Gas:  $\langle E_{\text{kin}} \rangle = 3/2 kT$
- $\Rightarrow U = 3/2 N k T = 3/2 nRT$

Use this to calculate specific heat:

$$C_v = \frac{1}{n} \frac{\Delta U}{\Delta T} = \frac{3}{2} R$$

## Isobaric ( $dP=0$ )

- $W, Q,$  and  $dU$  are all non-zero!

$$Q = \Delta U + P\Delta V$$

$$Q = nC_v\Delta T + P\Delta V = nC_v\Delta T + nR\Delta T$$

- Define “***molar specific heat at constant pressure***”:

$$C_p = \frac{Q}{n\Delta T} = C_v + R$$

Molar Specific Heat of a  
mono-atomic Ideal Gas at  
constant pressure

$$C_P = C_v + R = \frac{5}{2} R$$

# Adiabatic ( $Q=0$ )

- No heat exchange with environment

$$\Delta U + W = 0$$

- Define “**adiabatic exponent**”:

$$\gamma = \frac{C_p}{C_v}$$

# Adiabatic exponent of a mono-atomic Ideal Gas

$$\gamma = \frac{C_p}{C_v} = \frac{C_v + R}{C_v} = \frac{5}{3}$$

# Equipartition theorem

**When a system is in thermodynamic equilibrium, the average energy per molecule is  $1/2 kT$  for each degree of freedom.**

# Quantum Effects

- Not all degrees of freedom are accessible at low temperatures!
- =>  **$C_v$  thus changes as temperature increases and degrees of freedom “unfreeze”.**
- At low T: only translational dof=3.
- At medium T: also rotational dof=5.
- At high T: also rotations & oscillations dof=7.

# Note on di-atomic gases:

- At high temperatures internal energy  $U$  is not just in the kinetic energy of the molecules!
- E.g.  $H_2$ 
  - Below 100K  $\rightarrow$  only 3 dof
  - 100K-3000K  $\rightarrow$  5 dof , i.e. add 2 rotations
  - Above 3000K  $\rightarrow$  7 dof, i.e. add  $E_{kin}$  &  $E_{pot}$  for vibration.

# Adiabat in PV diagram

$$PV^\gamma = \textit{const}$$

$$TV^{\gamma-1} = \textit{const}$$

This is derived in Chapter 21 of the book, the “Math Toolbox” prior to example 21-3.

I expect you to be able to use this in problems !!!

# The 4 Processes

**TABLE 21-1** Ideal Gas Processes

	ISOTHERMAL	CONSTANT-VOLUME	ISOBARIC	ADIABATIC
<b>PV diagram</b>				
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You need to be able to analyze cyclic processes involving any of these 4 processes!

E.g. Diesel & Otto engine

Diesel: adiabat, isobar, adiabat, isochor

Otto: adiabat, isochor, adiabat, isochor

E.g. I describe two paths in the PV plane, and you tell me which one requires more heat, or does more work.

E.g.: Problem 61,62 and cumulative problem 1 in Chapter 22.