

Physics 2c

Lecture 2

Heat capacity & specific heat

Ideal Gas Law

Kinetic Theory of the Ideal Gas

Maxwell-Boltzmann distribution

Real Gases

# What is heat?

- Heat = **energy transferred** between objects as a result of a **temperature difference**.

# What is temperature?

- temperature = Measure of **average “internal energy”** of a many body system.

Heat Capacity:  $C$   
proportionality constant between  
heat  $Q$  and temperature  $T$

$$C = \frac{\Delta Q}{\Delta T}$$

Specific heat:  $c$   
heat capacity per unit mass

$$c = \frac{1}{m} C = \frac{1}{m} \frac{\Delta Q}{\Delta T}$$

# Heat Transfer

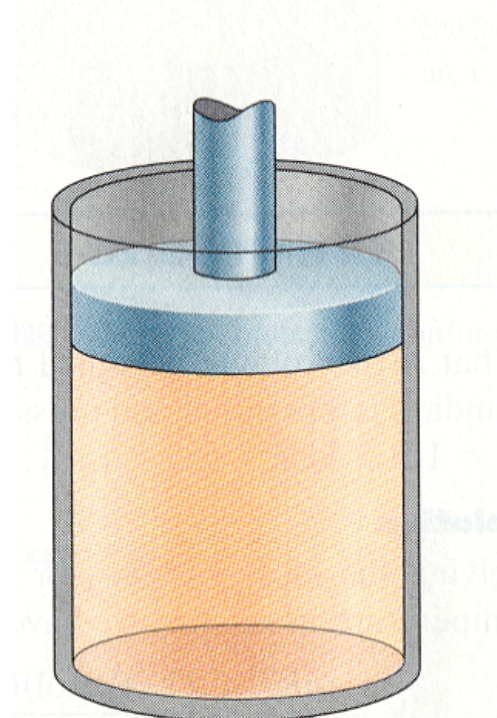
- Conduction: for things that “touch”
- Convection: for things that “flow”
- Radiation: for things that “radiate”

# Chapter 20: Ideal and Real gases

# Experimental observation

- $P \propto T$  if  $V = \text{const}$
- $P \propto 1/V$  if  $T = \text{const}$
- $V \propto T$  if  $P = \text{const}$

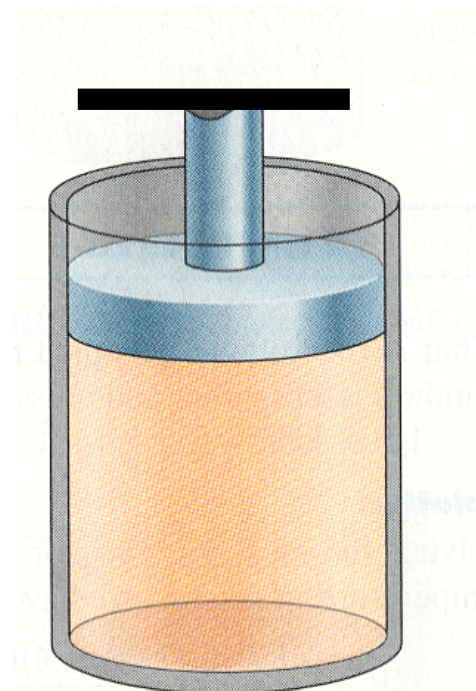
⇒  **$PV \propto T$**



True for most real gases  
under “typical” conditions

# How to keep $V$ constant

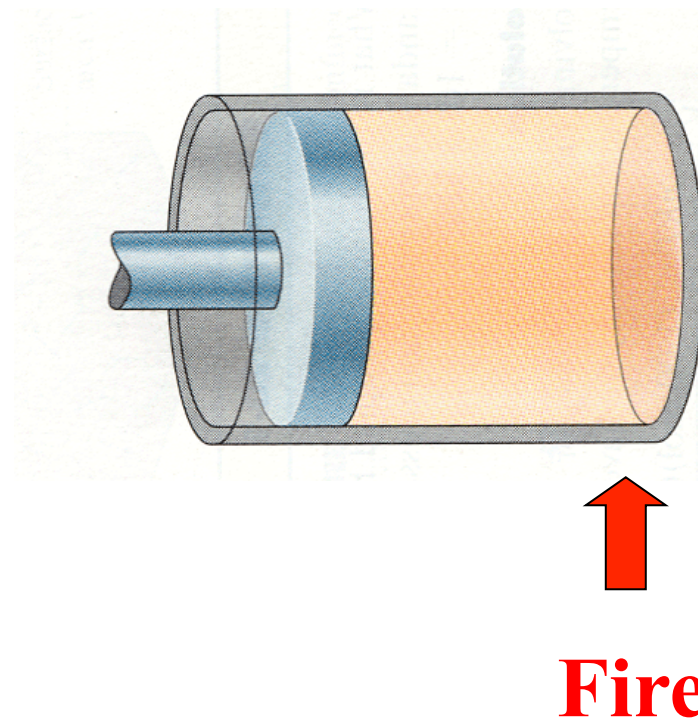
- Fix the piston
- Change  $T$  &  $P$  by putting flame underneath piston.



**Fire**

# How to keep $P$ constant

- Let piston move freely.
- Change  $T$  by putting flame underneath piston.
- Allow  $V$  to change freely.

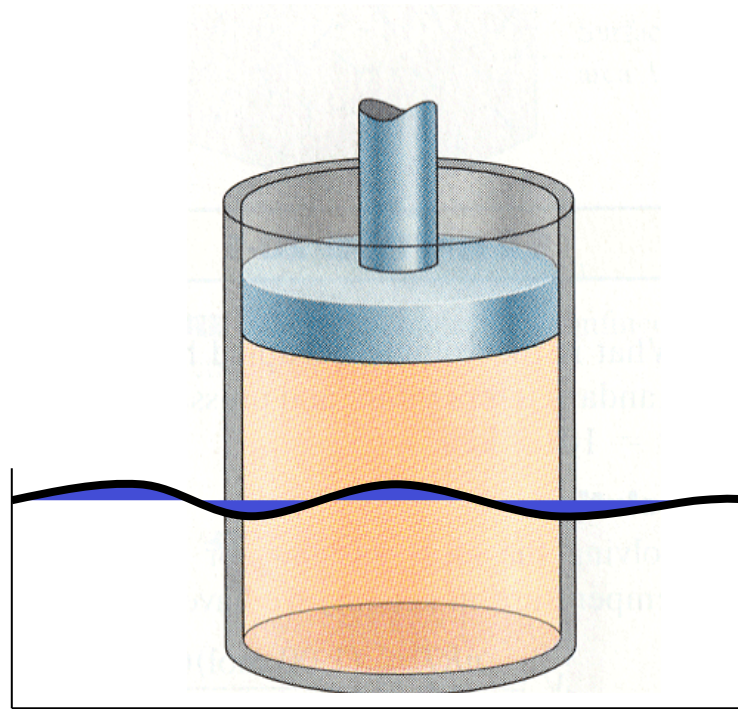




# How to keep $T$ constant

- Put piston in large thermal bath, e.g water.

If I push piston slowly in, gas and water remain in **thermal Equilibrium** while  $V$  decreases, and  $P$  increases.



# Ideal Gas Law

$$PV = N kT = nRT$$

**P = Pressure**

**V = Volume**

**N = number of gas molecules**

**T = temperature**

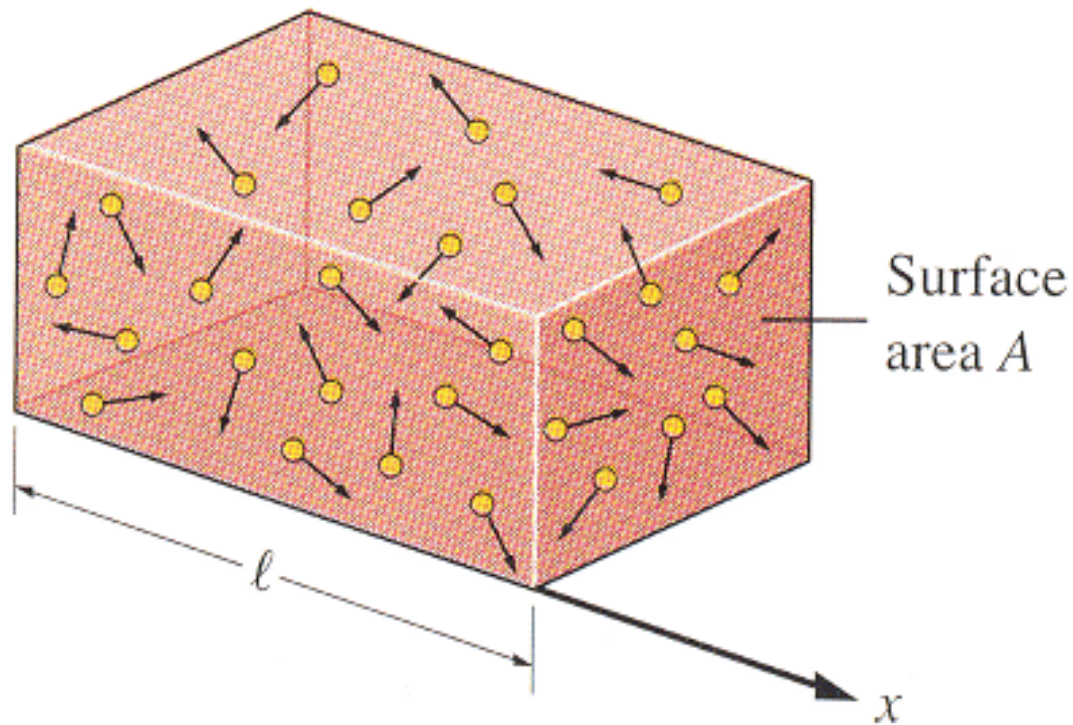
**k =  $1.38 \times 10^{-23}$  Joule/Kelvin = Boltzmann constant**

**n = number of moles of gas**

**R = N-Avogadro k = 8.314 J/(K mol)**

**Determined originally based on experimental observation.**

# Mental model for gas



**T = temperature**

**t = time**

**P = pressure**

**p = momentum**

**V = volume**

**v = velocity**

**N = number of particles**

**n = density of particles =  $N/V$**

**...**

**Nomenclature**

**I am using**

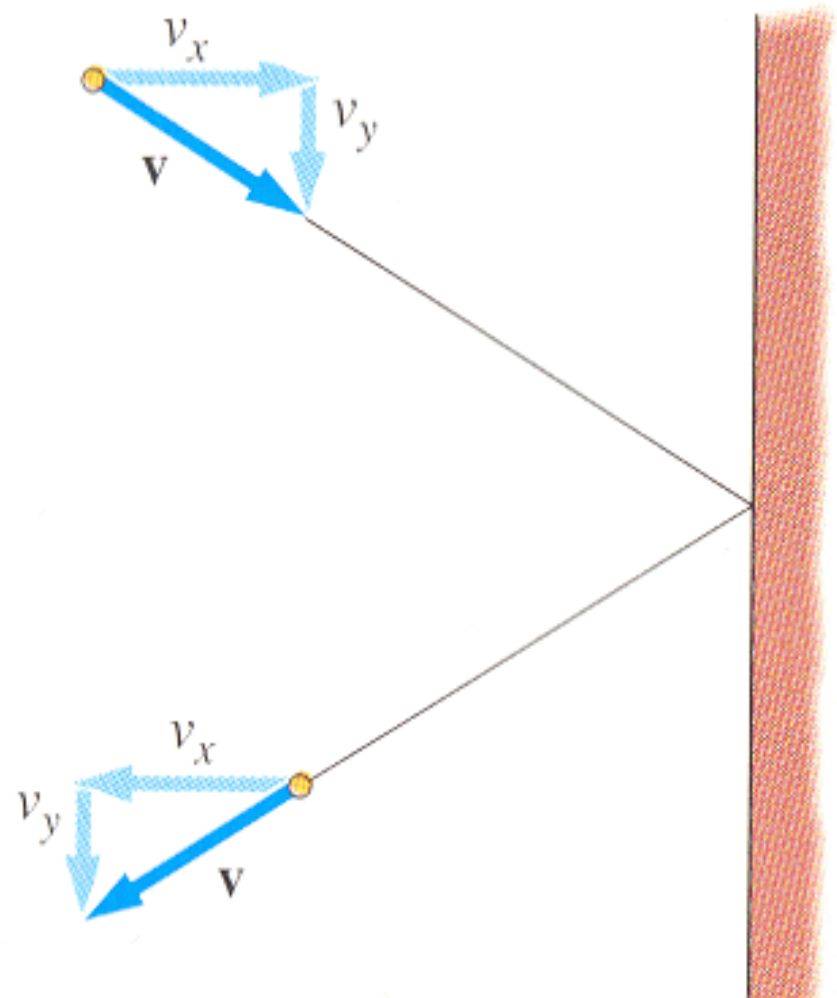
# Use mechanics to derive ideal gas law

Momentum difference =  $2 m v_x$

Force =  $dp/dt$

$dt = 2 \text{ length} / v_x$

Pressure = Force / Area



You do an ensemble average, and multiply by the number of particles.

# Consider a single particle

Momentum transfer to wall at bounce:  $\Delta p = 2mv_x$

Time between 2 bounces off same wall:  $\Delta t = \frac{2l}{v_x}$

Average force on that wall:  $F = \frac{\Delta p}{\Delta t} = \frac{mv_x^2}{l}$

Pressure on that wall  $P = \frac{F}{A} = \frac{mv_x^2}{Al} = \frac{mv_x^2}{V}$

For the pressure a single particle exerts on a confining wall in x-direction we thus get:

$$PV = m v_x^2$$

# Now, extend this to N particles

- $v_x^2$  becomes the average  $\langle v_x^2 \rangle$  averaged over all particles.

- This can then be compared to the average  $E_{\text{kin}}$

$$\langle E_{\text{kin}} \rangle = \frac{1}{2} m \langle v_x^2 + v_y^2 + v_z^2 \rangle$$

- Next assume that there is no preferred direction:

$$\langle E_{\text{kin}} \rangle = \frac{3}{2} m \langle v_x^2 \rangle$$

# Now compare:

- Microscopic derivation:

$$PV = \frac{2}{3} N \langle E_{\text{kin}} \rangle$$

- Macroscopic measurement:

$$PV = N kT$$

- We thus conclude that temperature is related to average kinetic energy of the ideal gas particles as:

$$\langle E_{\text{kin}} \rangle = \frac{3}{2} kT$$



# Kinetic Theory of Ideal Gas

Assumptions to derive Ideal Gas Law:

1. All collisions are elastic, conserving energy and momentum.
2. Movement of molecules is random. No preferred direction.
3. Large # of identical molecules of mass  $m$ , no structure, no size.
4. All energy in the gas exists in form of kinetic energy of its molecules.

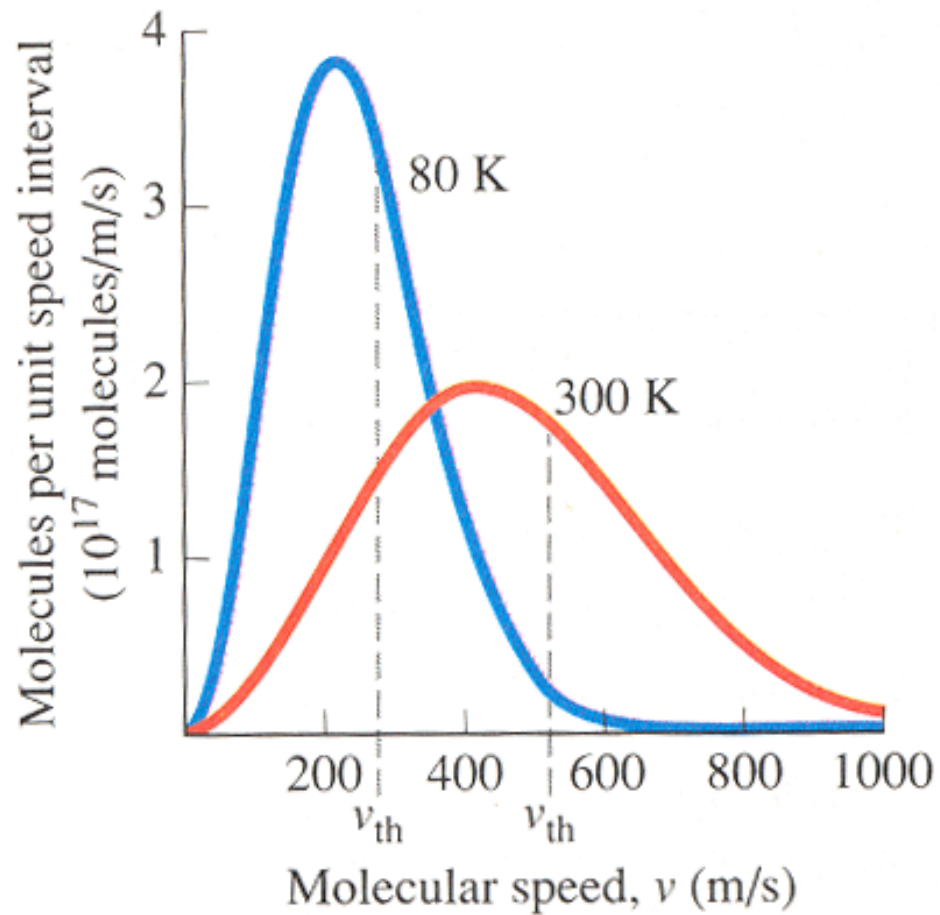
With this we derived the relationship between  $P, V$ , and  $\langle E_{kin} \rangle$  for an ideal gas in a rectangular box:

$$PV = \frac{2}{3} N \langle E_{kin} \rangle$$

by comparison with macroscopic definitions, we then concluded:

$$\langle E_{kin} \rangle = \frac{3}{2} kT$$

# Maxwell-Boltzmann Distribution



# Real Gases

$$(P + n^2 a / V^2) (V - n b) = n R T$$

Finite size of molecules  $\rightarrow$  b

Intermolecular forces  $\rightarrow$  a

**For small densities, n, both effects are negligible!**

# Equipartition theorem

**When a system is in thermodynamic equilibrium, the average energy per molecule is  $1/2 kT$  for each degree of freedom.**

# Aside on Equipartition Theorem

- Once you release the requirement of molecule being structureless you get:

$$\langle E_{\text{internal}} \rangle = f/2 kT$$

$f = \#$  of degrees of freedom of the gas molecules.