Physics 2c Lecture 12

Waves

Waves on a string

Wave power

Wave intensity

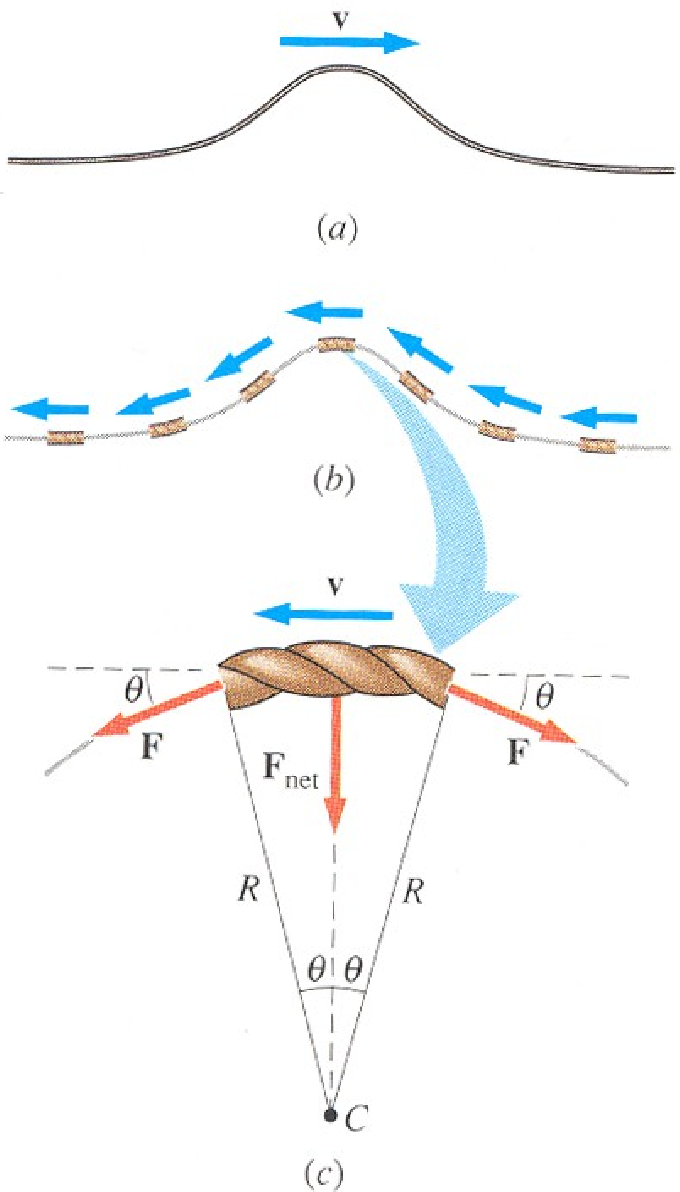
Example: Wave on a string

- We use our knowledge of mechanics to analyze physics of wave on a string.
- We make the simplifying assumption that movement of string near the top, i.e. at maximum amplitude, is circular motion.
- This gives us v of the wave, the rest is just math.

Introduce: $\mu = mass / length$

Aside on small angle approximation

- $\sin x = \tan x = x$ IFF $x << 1$
- We will be using this a few times for the remainder of this quarter.



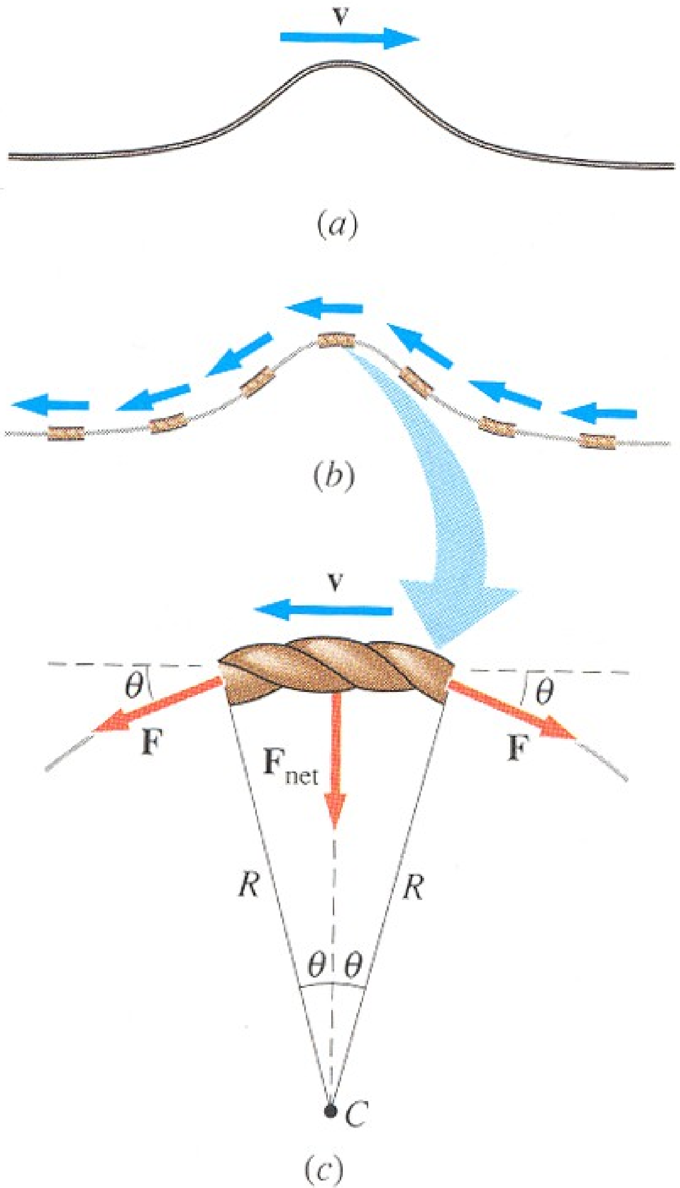
Waves on a string

$$F_{net} = 2F\sin\theta = 2F\theta$$

$$2F\theta = F_{zentripetal} = \frac{mv^2}{R}$$

$$\mu = \frac{m}{length} = \frac{m}{2R\sin\theta}$$

$$m = 2\theta R\mu$$



Waves on a string

$$2F\theta = \frac{mv^2}{R}$$

$$m = 2\theta R\mu$$

$$\Rightarrow 2F\theta = 2\theta\mu v^2$$

$$\Rightarrow v = \sqrt{\frac{F}{\mu}}$$

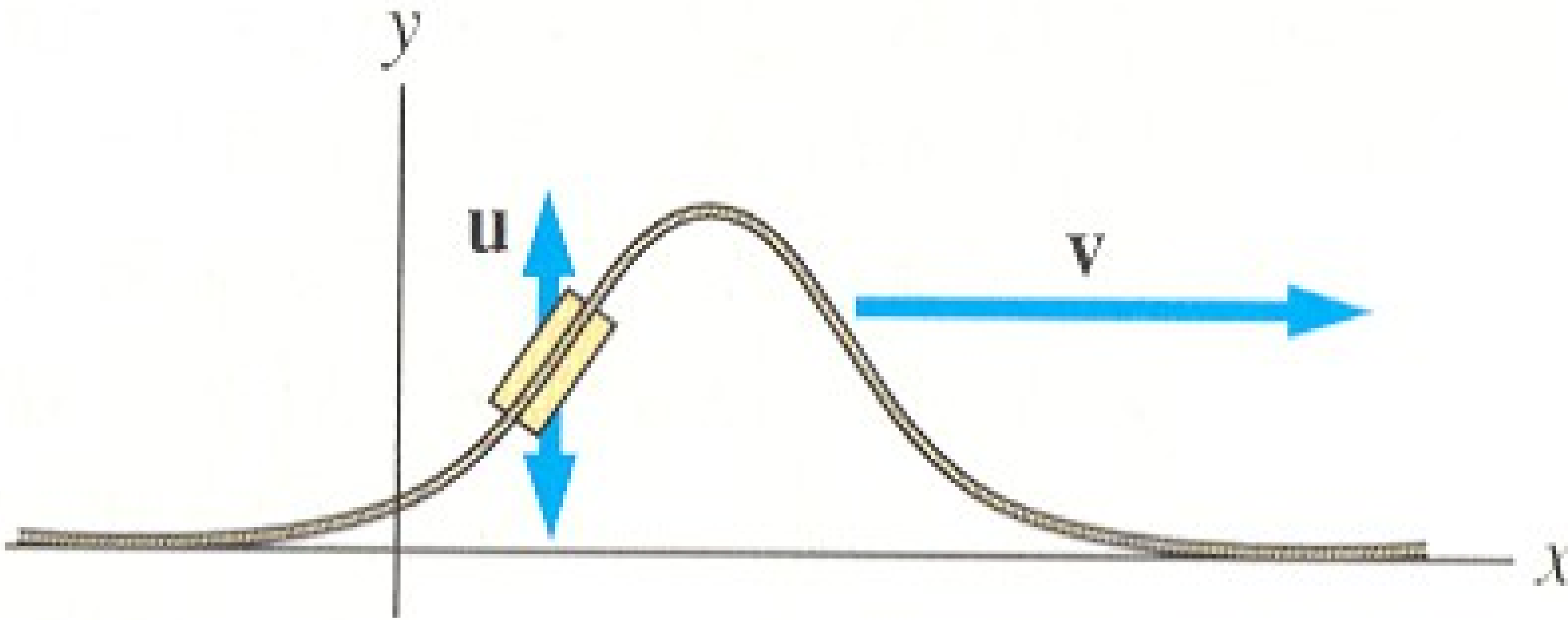
Note:

- The physics fixes the wave propagation velocity.
- The physics does not determine wavelength and period independently.
- Instead, a given medium will carry waves of many (all) periods.
- However, once you fix T the physics of the medium fixes   for you because it fixes v!

Wave power in a string

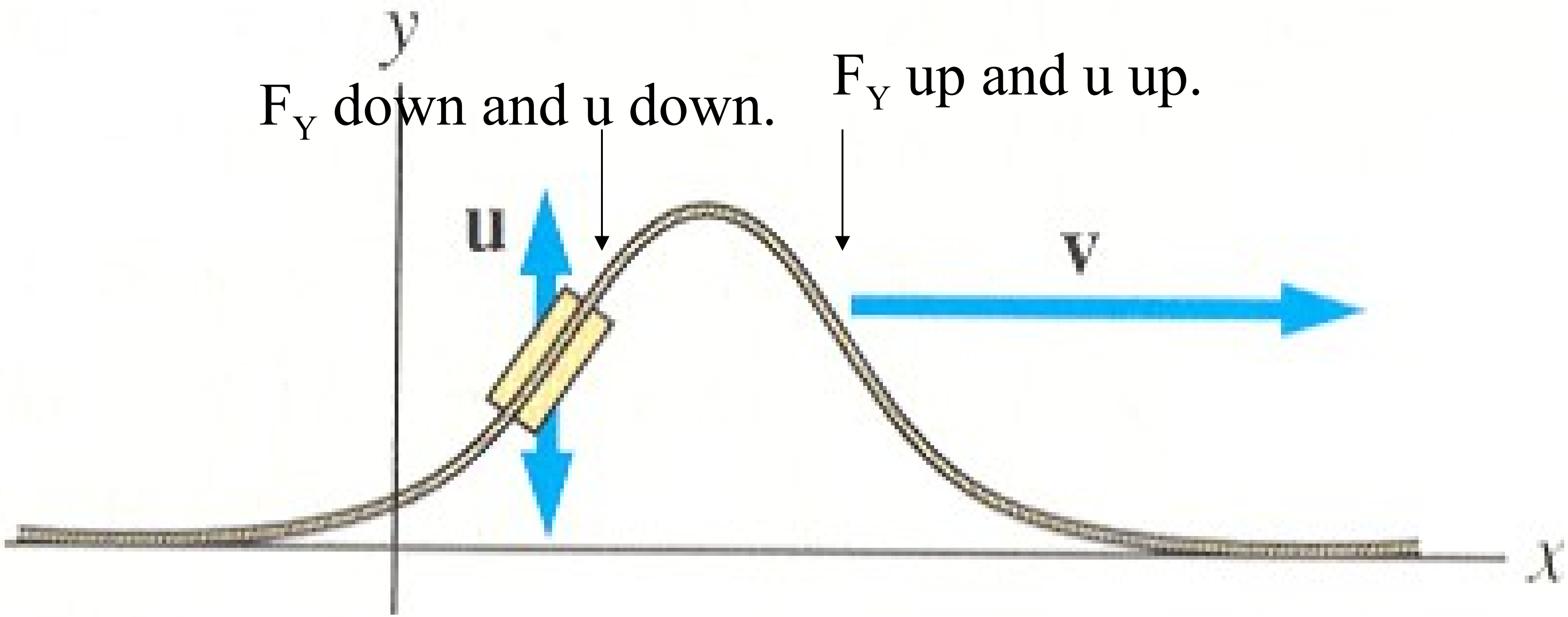
- Where is the energy in a wave?
  - It's in the displacement, y, of the medium.
  - It's distributed along the direction of wave motion. We'll thus be calculating an average power, averaged over x.
- Power = Energy/time = Force * speed
- Power = y-component of tension * y/ t

Aside: u != v



u "has" the power. v propagates the power.

F_Y parallel u at all times!



Wave Power

(derivation for point when F_y points towards negative y-axis)

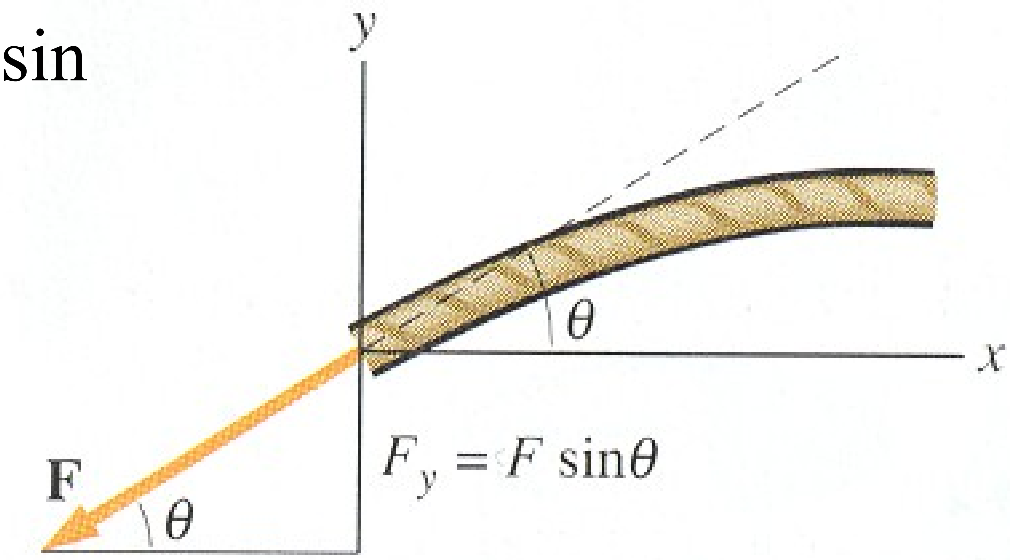
- Power = Force times velocity
- For string:

$$\text{Force} = F_y = -\ \text{Tension} * \sin$$

$$\text{velocity} = \frac{\partial y}{\partial t}$$

- Small angle approx:

$$\sin\theta \cong \tan\theta = \frac{\partial y}{\partial x}$$



Wave Power - putting it all together

$$y(x,t) = A\cos(kx - \omega t)$$

$$u = \frac{\partial y}{\partial t} = \omega A \sin(kx - \omega t)$$

$$\sin\theta = \tan\theta = \frac{\partial y}{\partial x} = -kA\sin(kx - \omega t)$$

$$P = -F \cdot u \cdot \sin\theta = F\omega k A^2 \sin^2(kx - \omega t)$$

$$\langle P \rangle = \frac{1}{2} F\omega k A^2 = \frac{1}{2}\mu\omega^2 A^2 v$$

Where we use:

$$k = \omega / v$$

$$F = \mu v^2$$

Aside on signs in the game

$$y(x,t) = A\cos(kx - \omega t)$$

$$u = \frac{\partial y}{\partial t} = \omega A \sin(kx - \omega t)$$

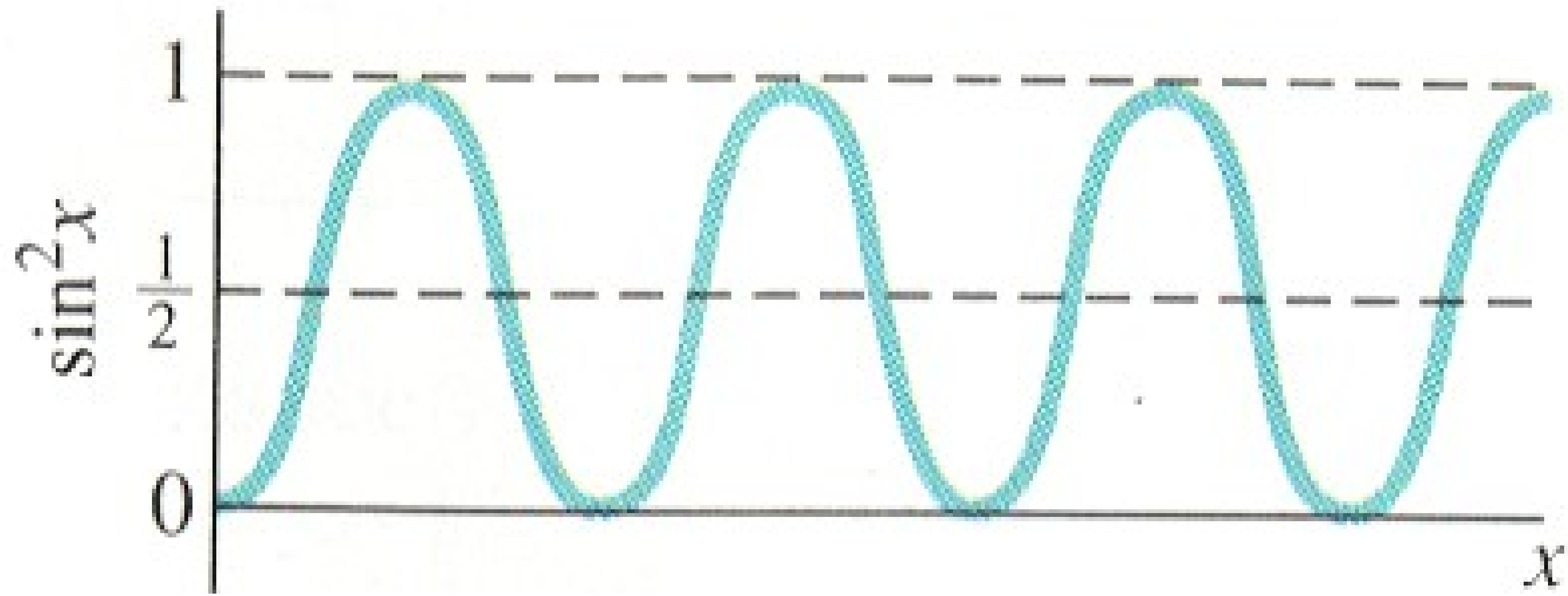
$$\tan\theta = \frac{\partial y}{\partial x} = -kA\sin(kx - \omega t)$$

$$P = -F \cdot u \cdot \sin\theta = F\omega k A^2 \sin^2(kx - \omega t)$$

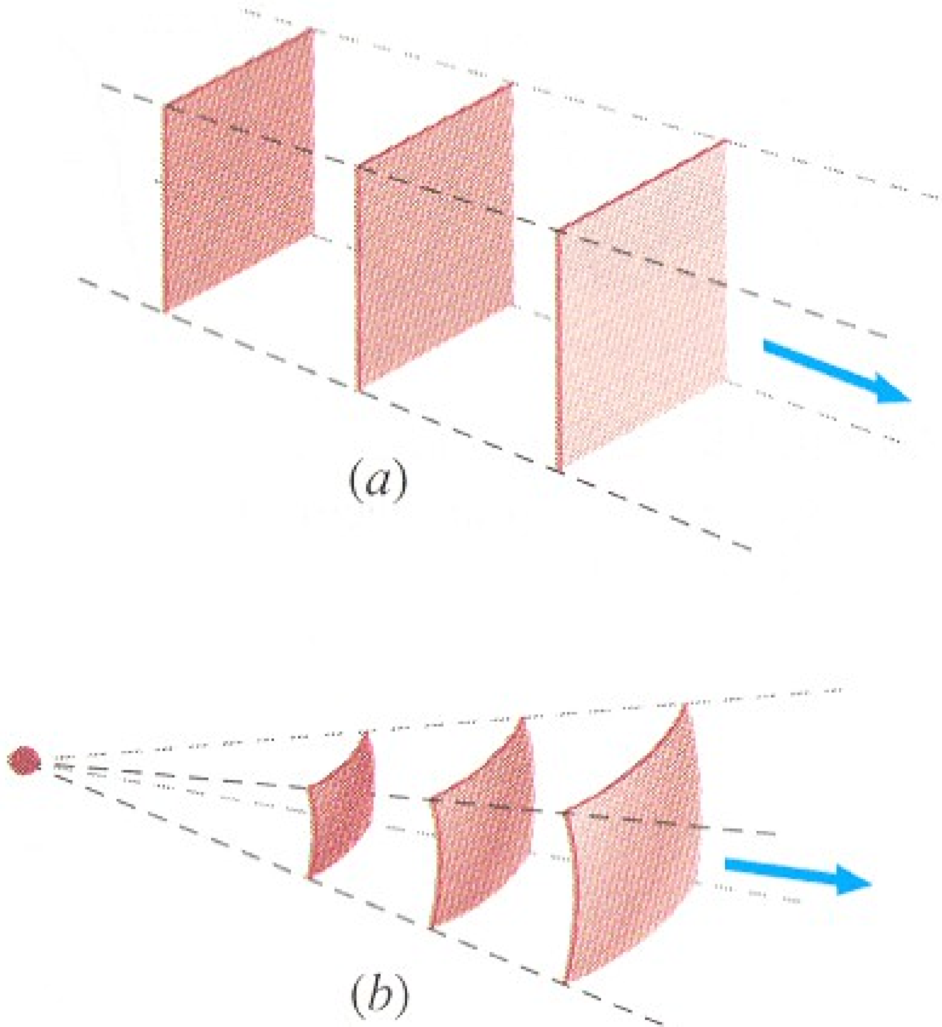
P is always positive!

Works out because F,k, >0

Aside on averaging



Wave Intensity = Power/area



- Average rate at which the wave carries energy across a unit area surface perpendicular to the wave propagation.

Question

- 2km away from the source of a spherical wave a wave intensity I is measured.
- What’s the wave intensity 4km away?

(a) Also I

(b) I/2

(c) I/4

Example: Spherical wave

$$I = \frac{Power}{Area} = \frac{Power}{4\pi R^2}$$