Physics 214 UCSD

Lecture 5

- Symmetries & QCD
  - Finishing off Isospin et al.
  - SU(3)
- Draw heavily from H&M chapter 2 today.

SU(3)

- Start with general characteristics
  - Generators and fundamental representation
  - T,U,V spin; SU(2) embedded in SU(3)
  - Graphical way to construct multiplets
  - Applications:
    - Flavor SU(3)
    - Color SU(3)

SU(3) Generators

$3^2-1 = 8$ generators λ_i :

$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix};\begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix};\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}$$

$$\begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix};\begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix};\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

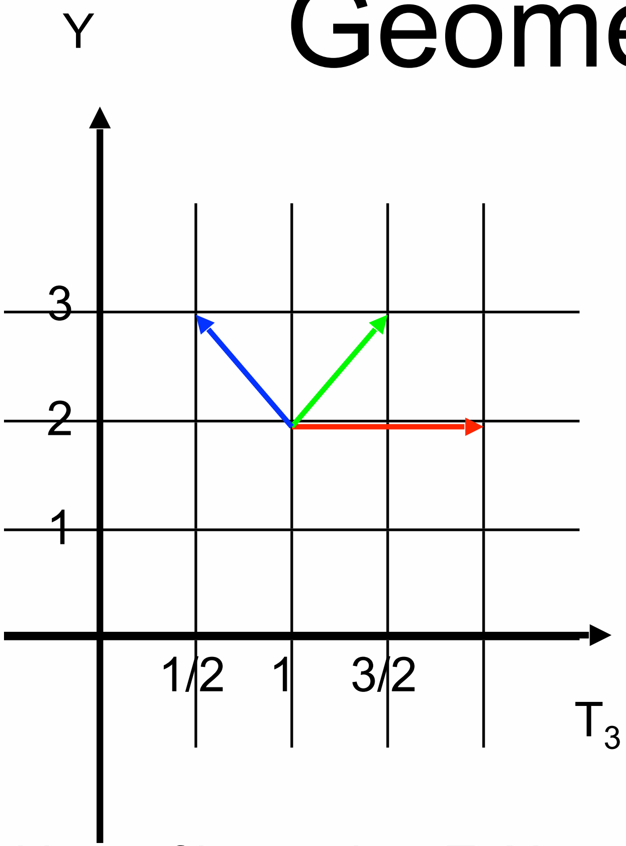
$$\begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix};\sqrt{\frac{1}{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$$

Interesting structure in that there are three spin-1/2 subspaces.

Rank = 2
⇒ D(a,b) to classify multiplets.
⇒ T_3 and Y quantum numbers within multiplet.

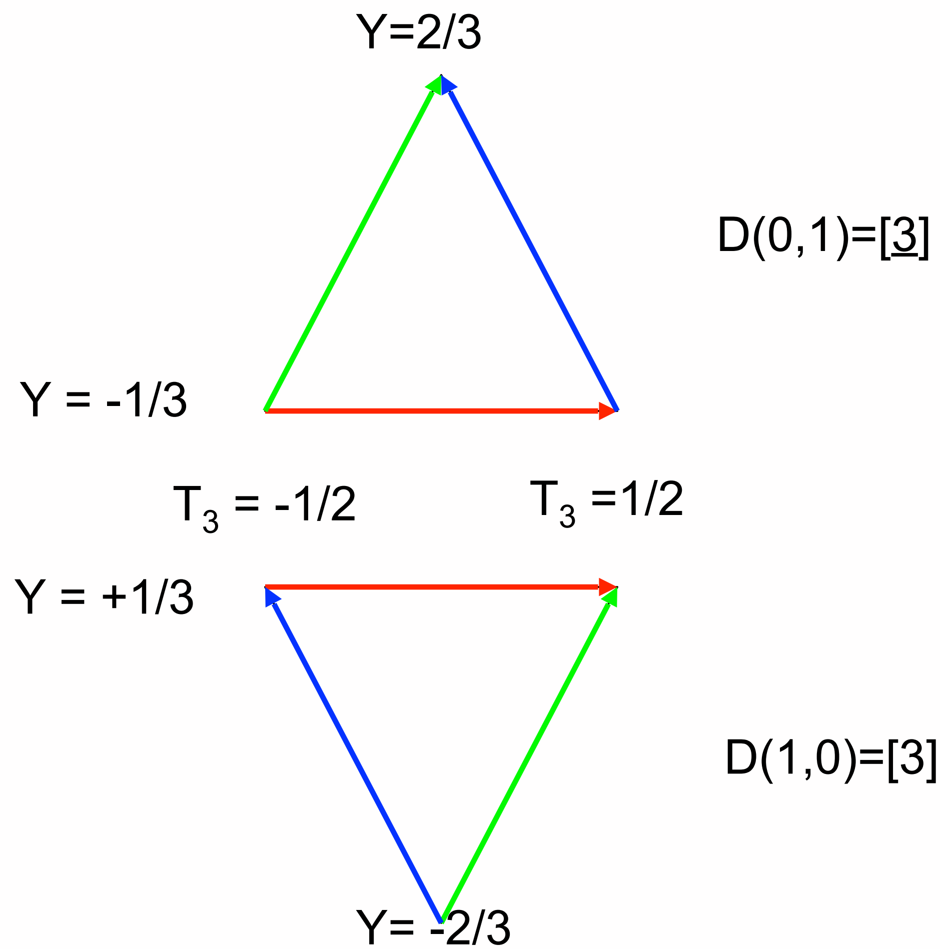
$$T_3 = \lambda_3 / 2$$
$$Y = \lambda_8 / \sqrt{3}$$

Geometric Construction



Lines of increasing: T_3,Y

Fundamental Representations:



Isospin: T_+ (m,y) = (m+1,y)

U-spin: U_+ (m,y) = (m-1/2,y+1)

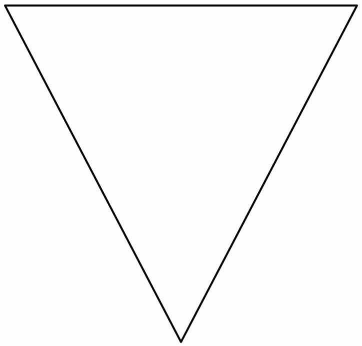
V-spin: V_+ (m,y) = (m+1/2,y+1)

Convention: D(width at top, width at bottom)

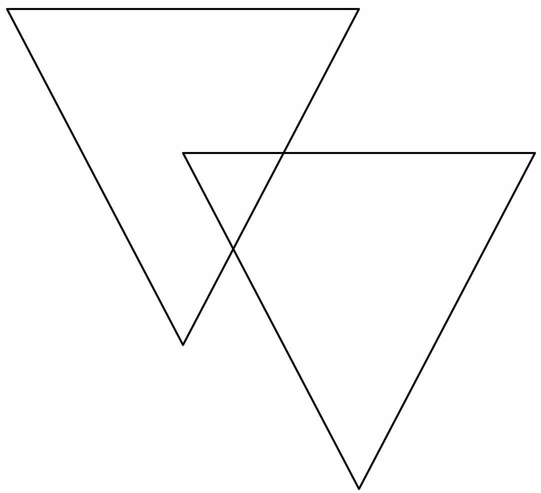
Three additional rules:

- For a given multiplet, the outer most ring can only occupy one state at each point.
- Going inwards, you get multiple occupation per site that belong to the same multiplet.
  - Going one ring in, add one more state per site on that ring.
- If you get more points than would fit on that ring at that site, then collect left-overs, because they will form a separate multiplet.

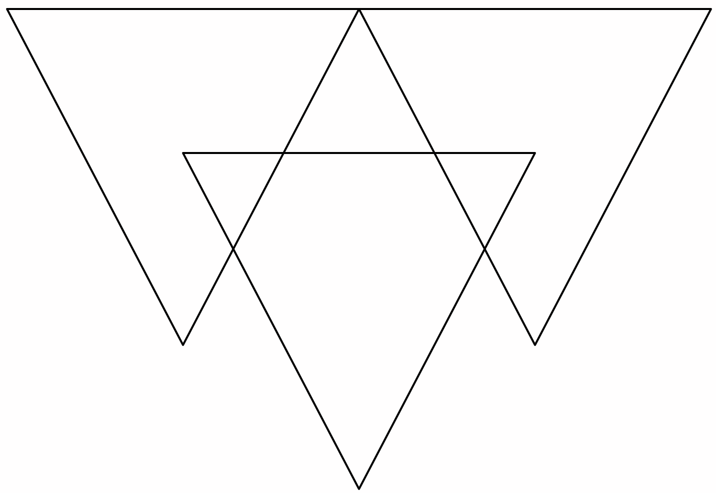
Constructing 3 x 3



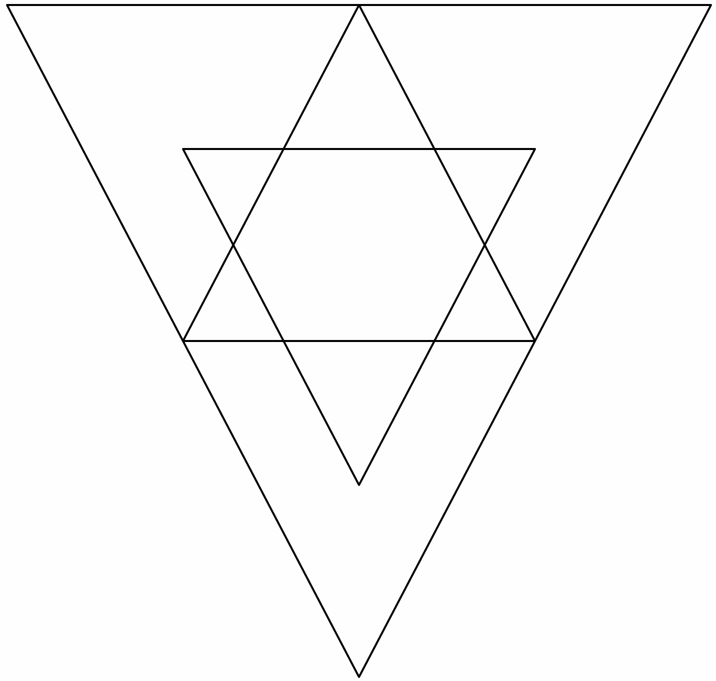
Constructing 3 x 3



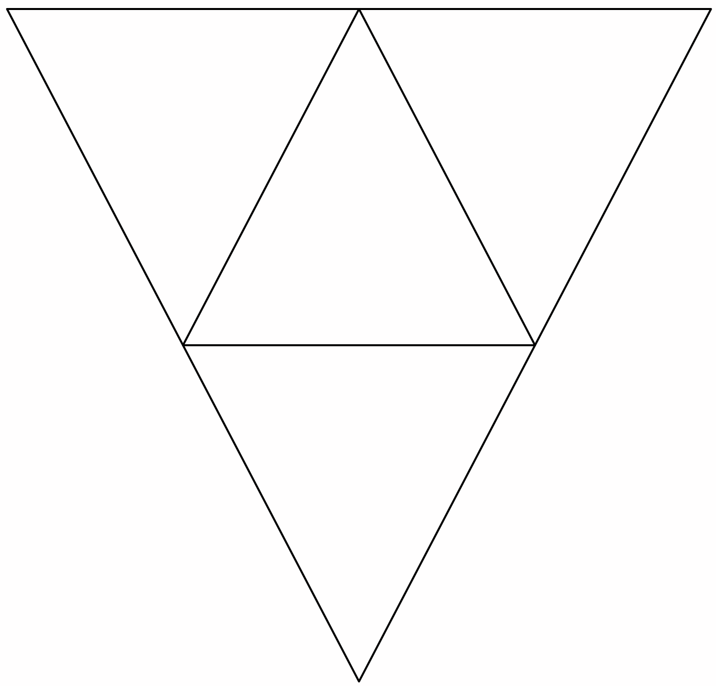
Constructing 3 x 3



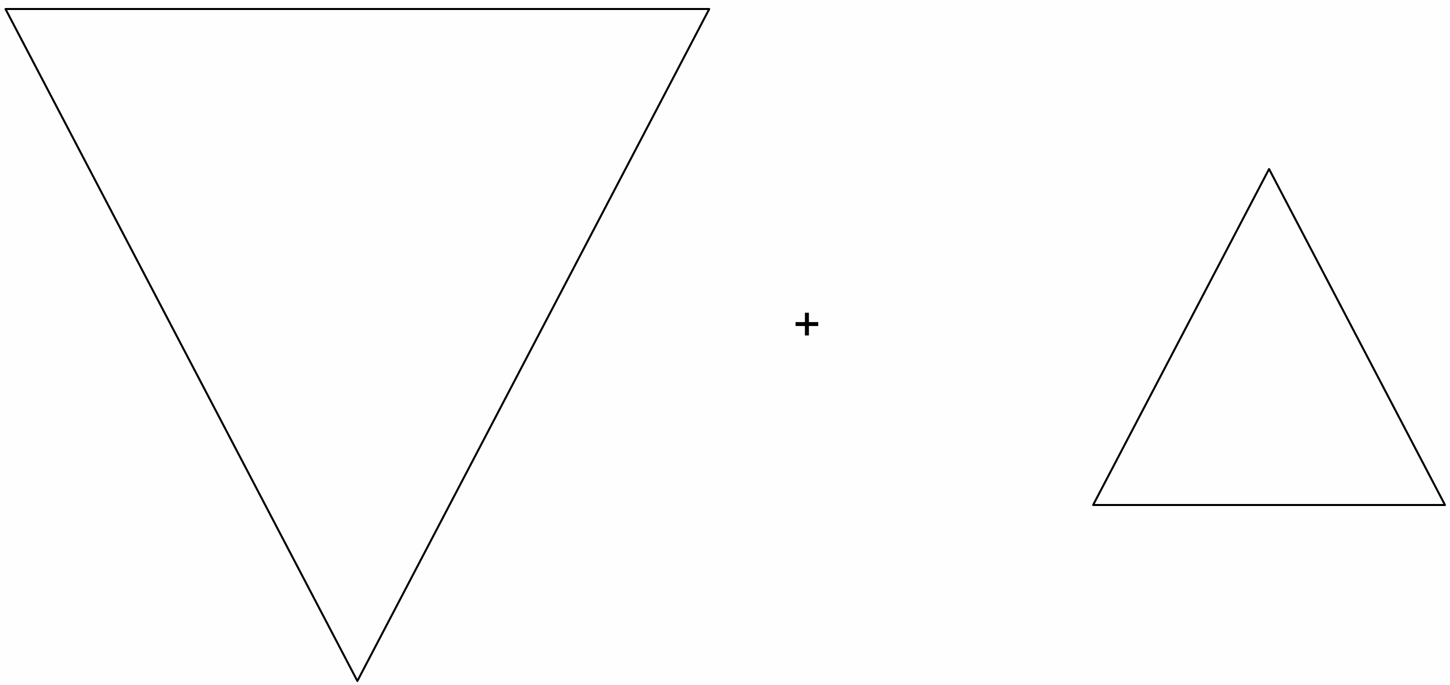
Constructing 3 x 3



Constructing 3 x 3

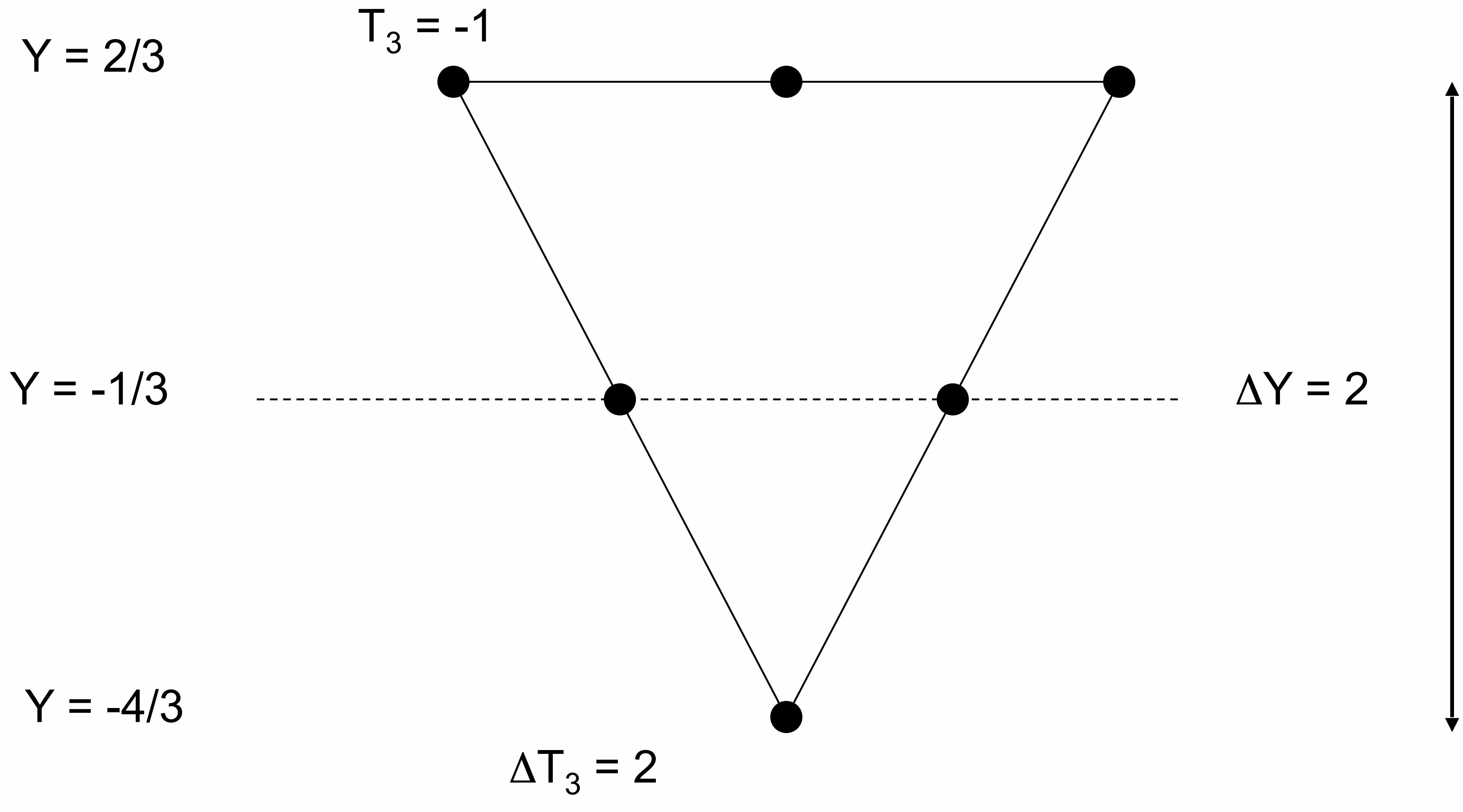


$3 \times 3 = 6 + \underline{3}$



This is a sextuplet, or in group notation
D(2,0) = [6]

D(width at top, width at bottom)



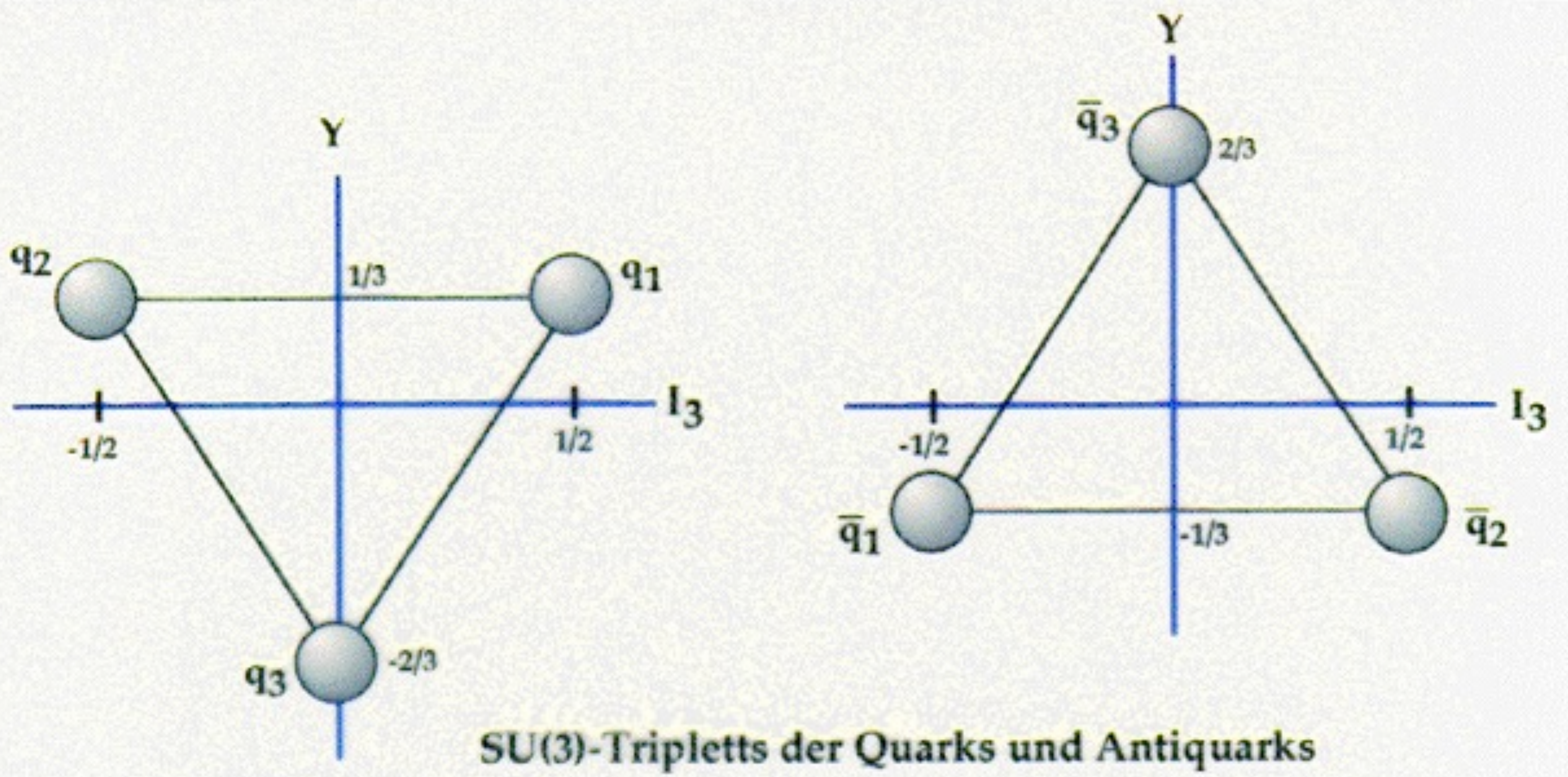
All multiplets of SU(3) can be constructed in this fashion.

Significance to Physics
Flavor SU(3)

- Identify:
    - T = isospin
    - Y = B + S = baryon number + strangeness

    $\Rightarrow$ Charge = Q = T_3 + Y/2

- Identify 3 with quarks *u,d,s* and $\underline{3}$ with antiquarks $\underline{u}$, $\underline{d}$, $\underline{s}$
- Not all SU(3) multiplets are physically meaningful !!!
    – A physical state needs to simultaneously satisfy SU(3) flavor and SU(3) color, and has to have the appropriate overall symmetry under interchange.
    – The two symmetries operate on completely separate Hilbert spaces. The fact that both are SU(3) is an accident of nature.



SU(3)-Tripletts der Quarks und Antiquarks

$d = q_2 = (-1/2,\ 1/3)\quad Q = -1/2 + 1/6 = -1/3$

$u = q_1 = (+1/2,\ 1/3)\quad Q = +1/2 + 1/6 = +2/3$

$s = q_3 = (0, -2/3)\quad Q = 0 - 1/3 = -1/3$

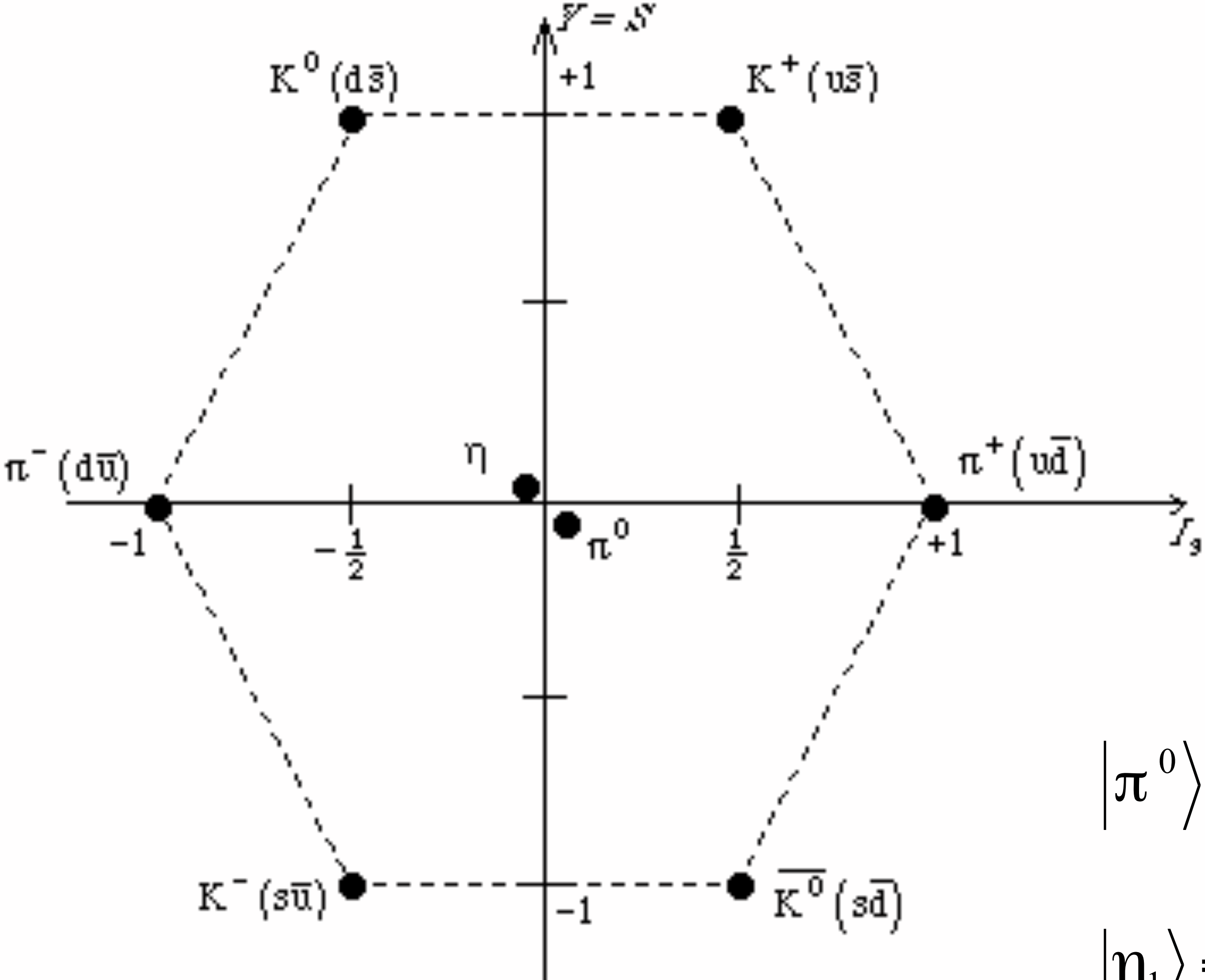
Examples:

- Mesons: $3 \times \underline{3} = 8 + 1$
  - Works for ground state as well as excited states.
- Baryons: $3 \times 3 \times 3 = (6 + \underline{3}) \times 3$

$$= 10 + 8 + (3 \times \underline{3})$$

$$= 10 + 8_S + 8_A + 1_A$$

- Note: The $\underline{3}$ in $(6 + \underline{3})$ is different from the quark triplet. It is the antisymmetric di-quark triplet.

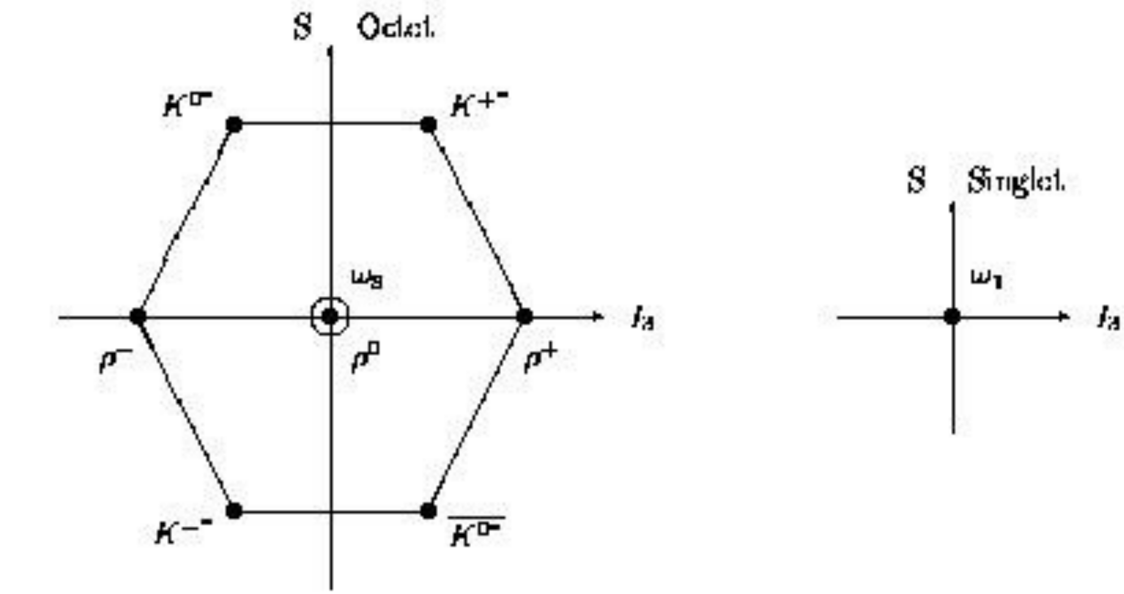


$$\left|\pi^0\right\rangle = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$$

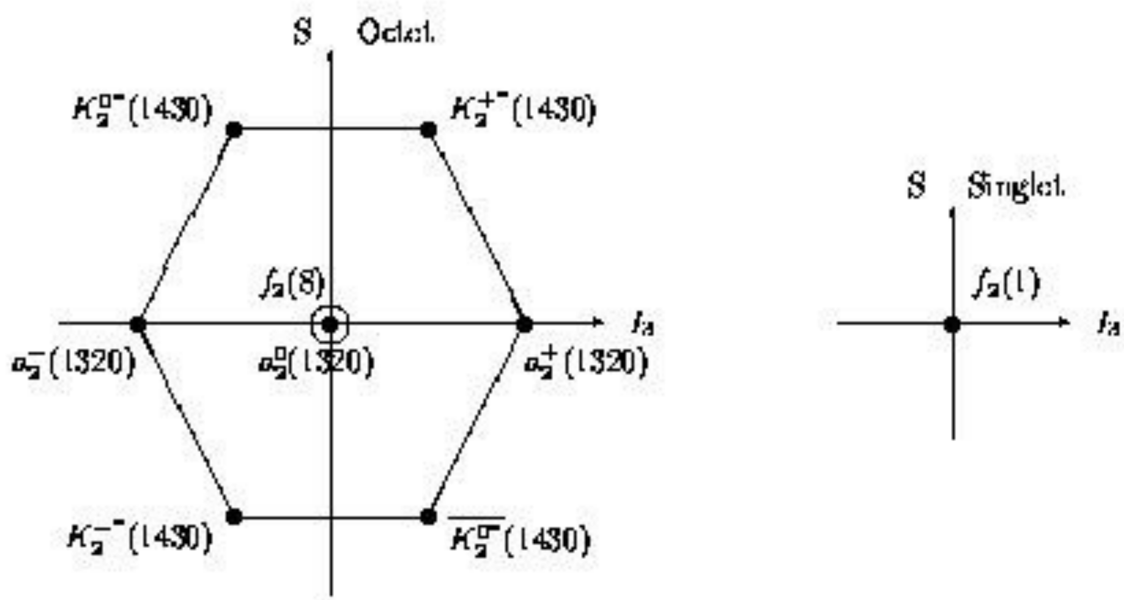
$$\left|\eta_1\right\rangle = \frac{u\bar{u} + d\bar{d} + s\bar{s}}{\sqrt{3}}$$

$$\left|\eta_8\right\rangle = \frac{u\bar{u} + d\bar{d} - 2s\bar{s}}{\sqrt{6}}$$

The vector mesons $J^{PC} = 1^{--}$



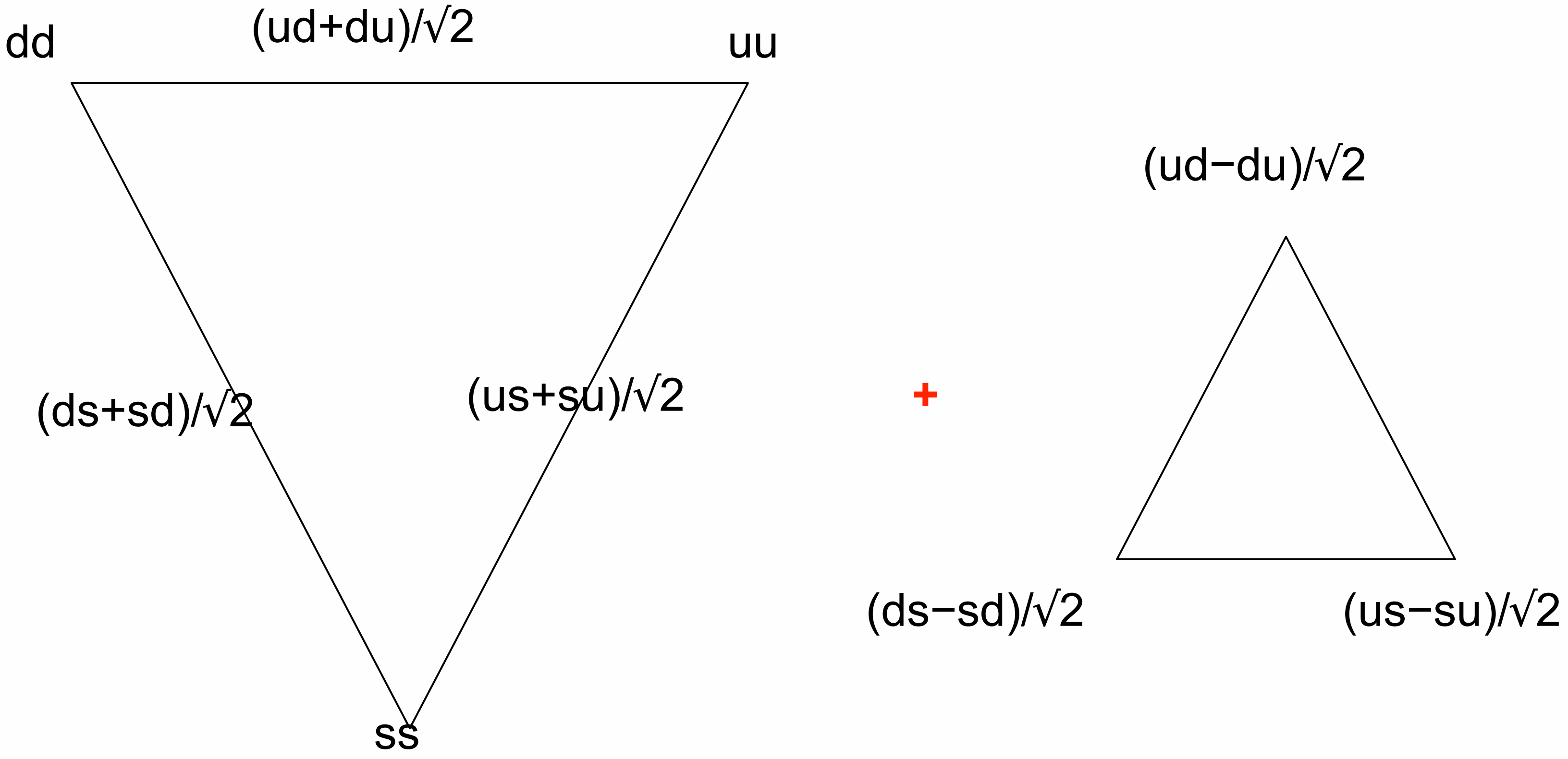
The tensor mesons $J^{PC} = 2^{++}$



Aside:

- SU(3) flavor is strongly dynamically broken in nature.
  - Masses within a multiplet depend on *s*-quark content.
  - Physical states with $T = 0$ mix across singlet and octet. For vector mesons the physical states are indeed flavor orthogonal rather than flavor symmetric.
- Flavor SU(3) most important to order particles into multiplets, and to show that color must exist, and have (at least some) SU(3) properties.

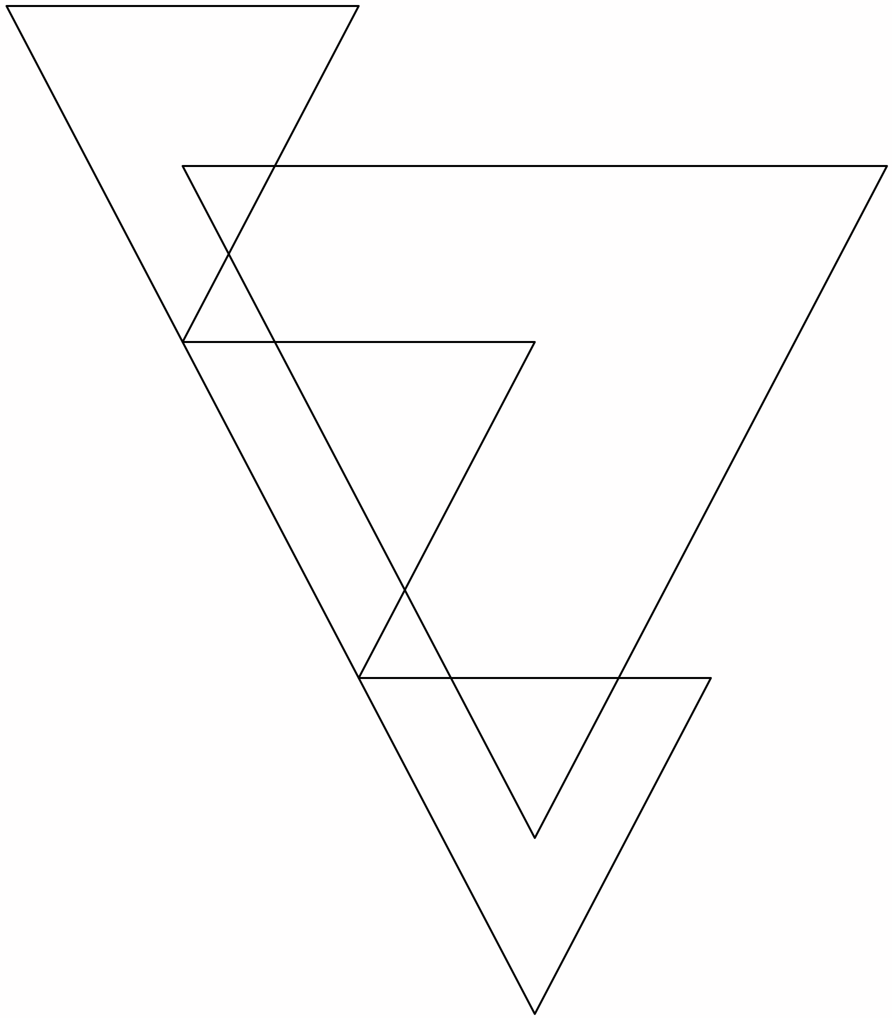
3 x 3 = 6 + 3 for di-quarks



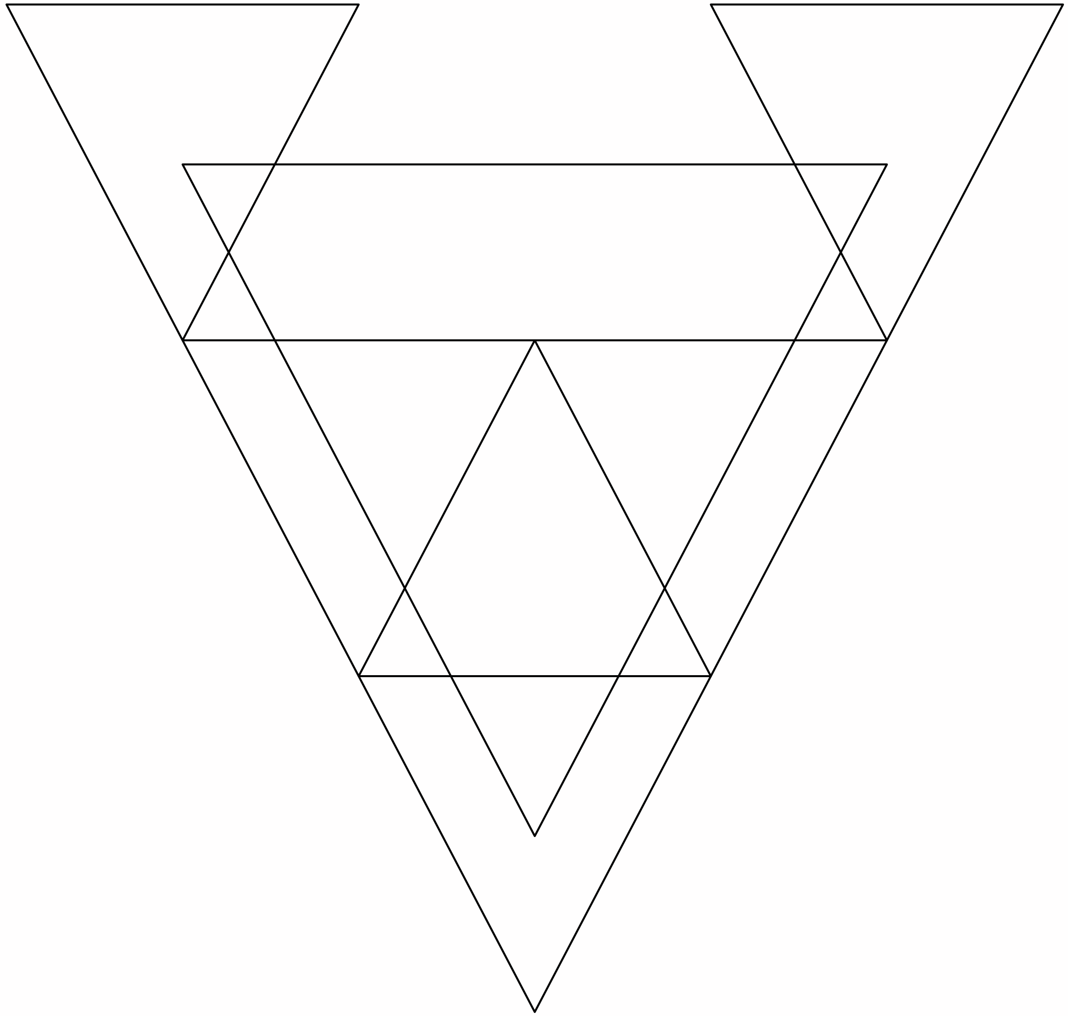
Symmetric Sextet

Antisymmetric triplet

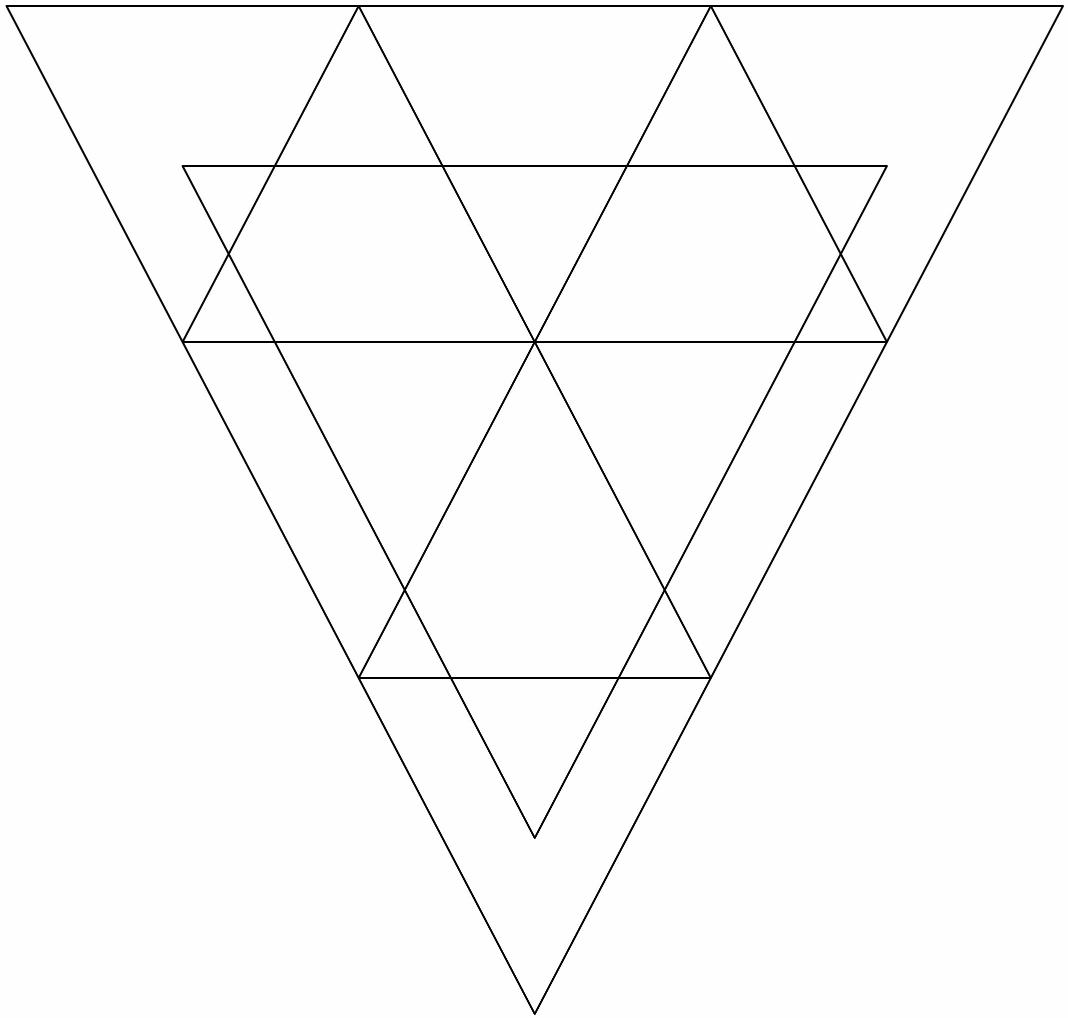
Symmetric di-quarks to triquarks



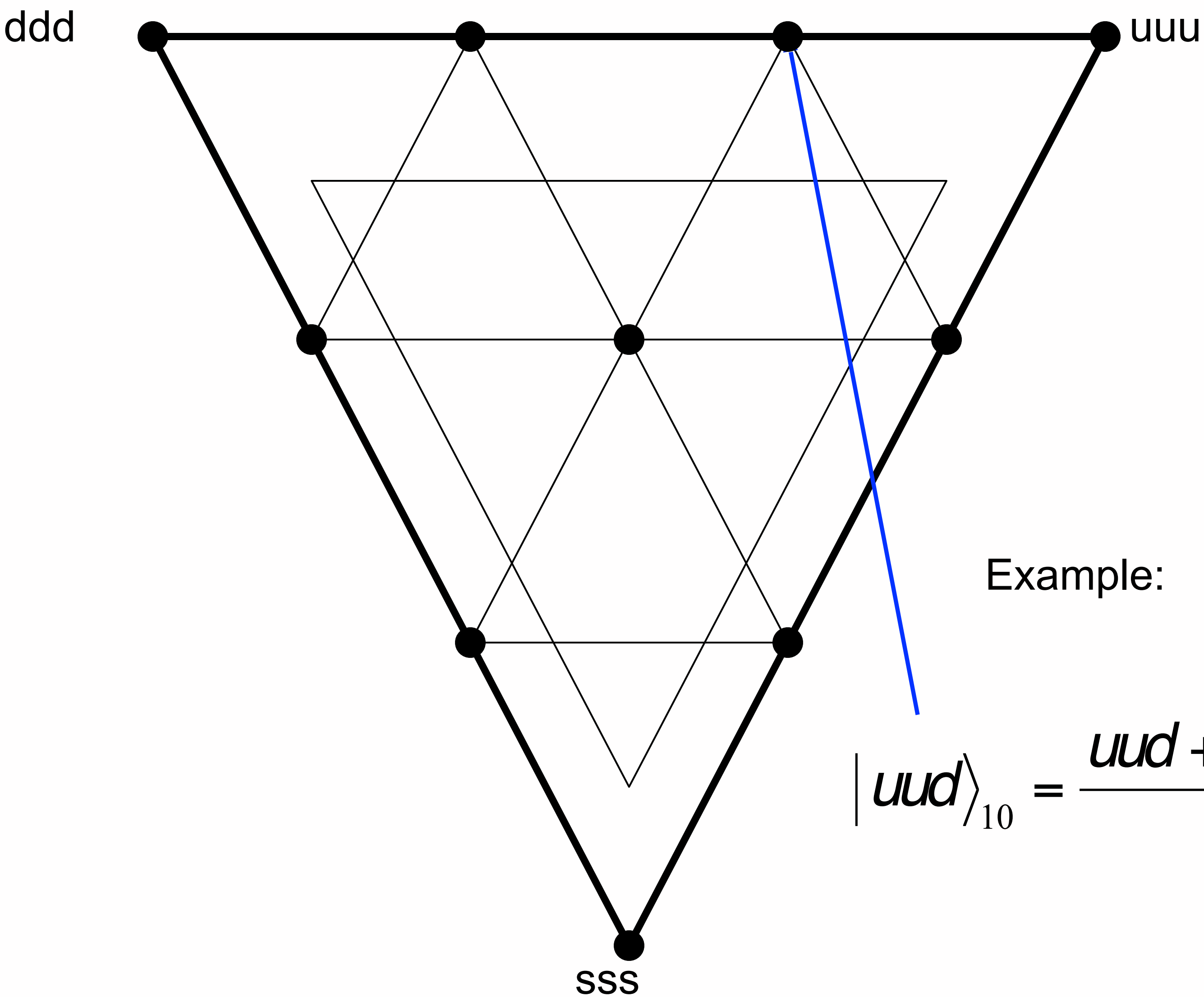
Symmetric di-quarks to triquarks



Symmetric di-quarks to triquarks



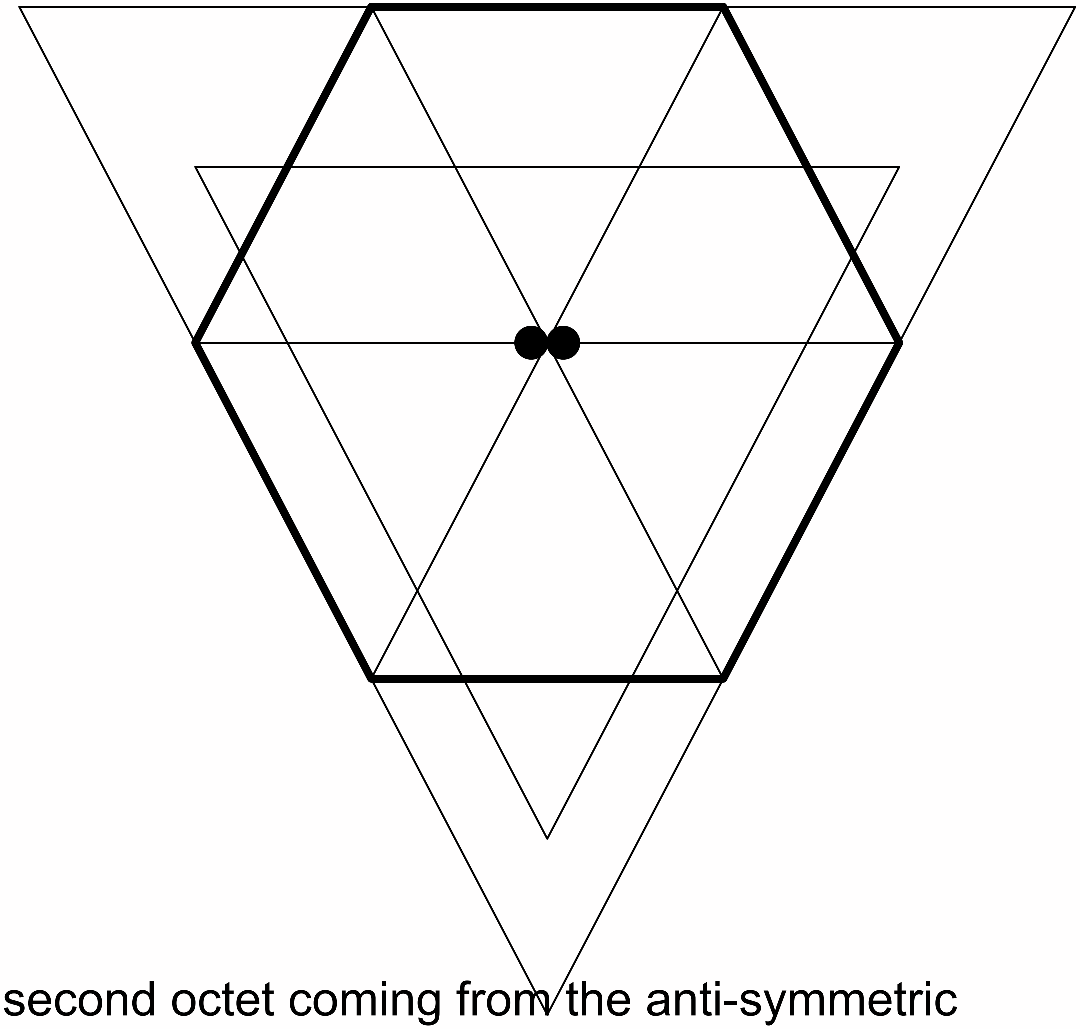
Symmetric 10-plet



Example:

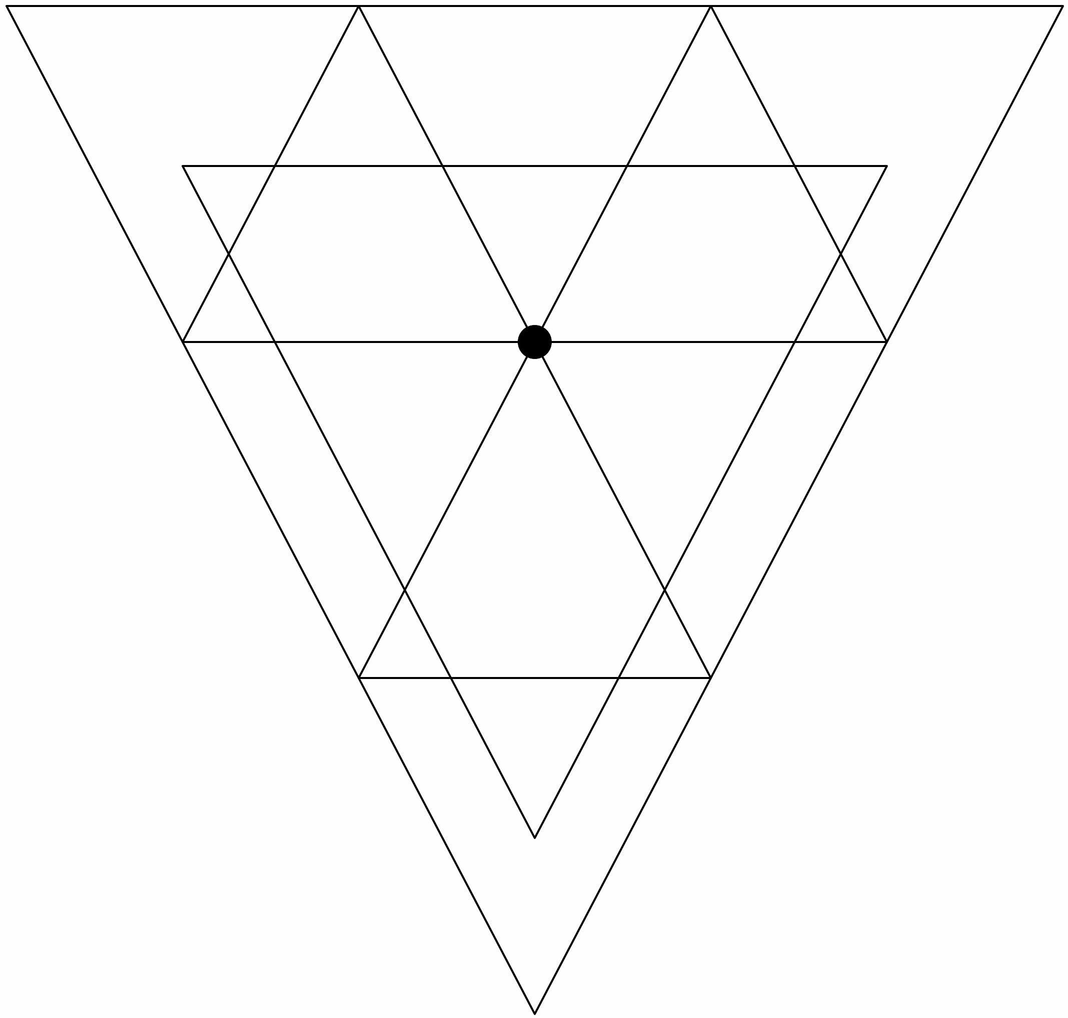
$$\left| uud \right\rangle_{10} = \frac{uud + udu + duu}{\sqrt{3}}$$

Octet with symmetric di-quarks



There’s a second octet coming from the anti-symmetric
Di-quark triplet x quark triplet.

Singlet (totally anti-symmetric)



Conclusion from flavor SU(3) alone:

- We could have as ground state baryon multiplets:
  - One fully symmetric decuplet
  - Two octets
    - One is symmetric for the di-quarks
    - The other asymmetric for the di-quarks
  - One singlet

Next, we show that the lowest lying baryons are classified in
one octet and one decuplet
due to spin-statistics on the total wave function.

Total wave function symmetry must be anti-symmetric with interchange of any two fermions

(Spin-statistics theorem)

- (baryon) = (flavor) (spin) (space) (color)
- (color) = antisymmetric singlet of SU(3)

  ⇒(flavor) (spin) (space) = symmetric
- Spin:
  - $2 \times 2 \times 2 = (3_s + 1_A) \times 2 = 4_s + 2_{s12} + 2_{A12}$
    - i.e. one spin 3/2 and two spin 1/2 multiplets possible.
- Ground state ⇒ L = 0, symmetric for interchange of identical quarks.
- Need to combine symmetric states from spin and flavor, *i.e.,* not all combos valid.

12 refers to interchange of quark 1 and quark 2, i.e di-quark symmetry under interchange.

Symmetric Flavor-spin combos

Flavor SU(3)

- 10 = symmetric
- 8 = S12
- 8 = A12
- 1 = anti-symmetric

Spin SU(2)

- 4 = symmetric
- 2 = S12
- 2 = A12

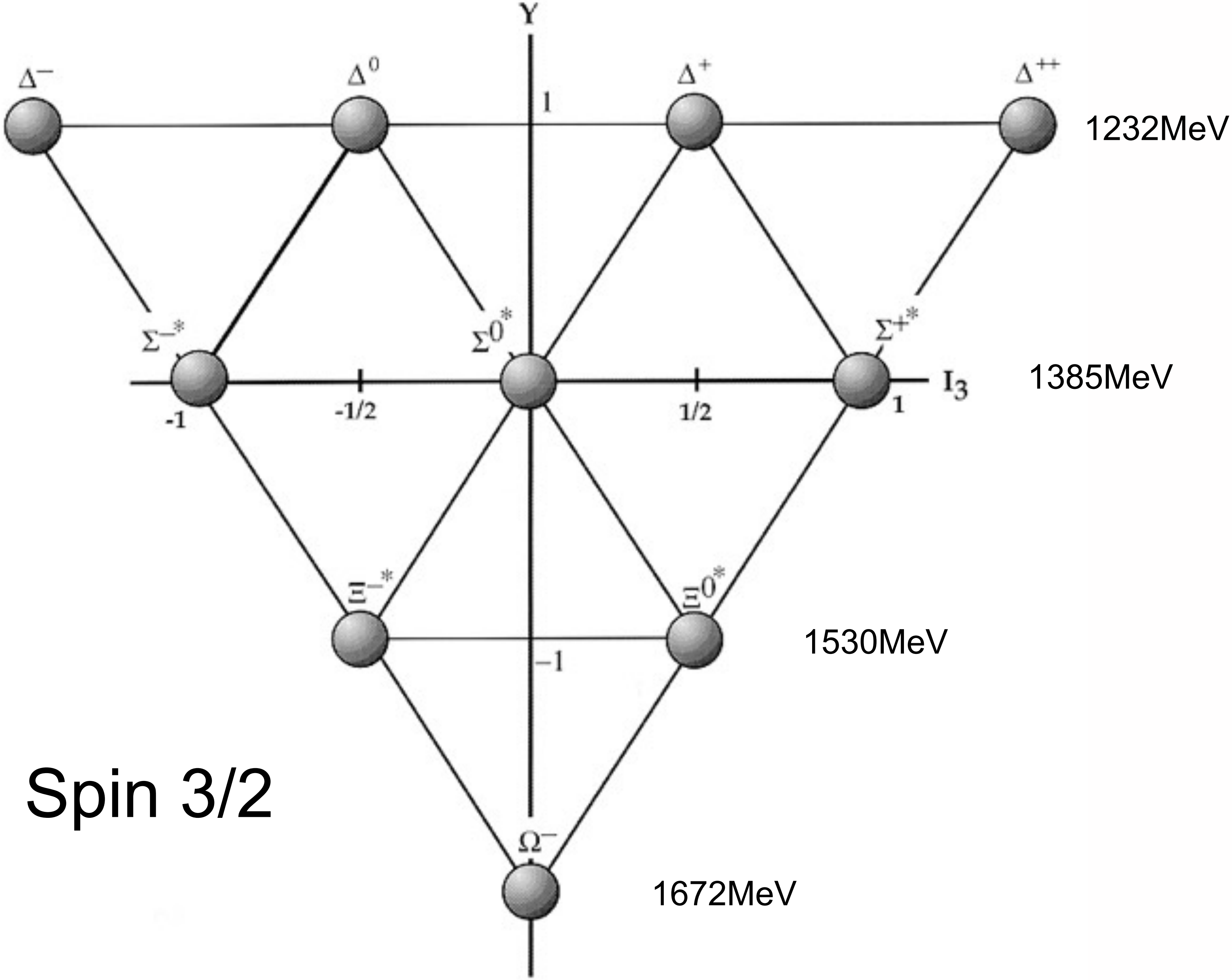
possible options are thus:
Spin 3/2 decuplet, *i.e.* (flavor,spin) = (10,4)
Spin 1/2 octet symmetric 1↔2, i.e. $(8_{S12},2_{S12})$
Spin 1/2 octet antisymmetric 1↔2, i.e. $(8_{A12},2_{A12})$

E.g., the symmetric flavor-spin octet is thus:
$(1/\mathrm{sqrt}(2)) * [(8_{S12},2_{S12}) + (8_{A12},2_{A12})]$

Color is necessary

- If color did not exist then the Flavor SU(3) Decouplet would have to be combined with the anti-symmetric spin 1/2 doublet in order to make the total wave function anti-symmetric.
- This would predict the uuu, ddd, sss baryons to have spin 1/2 instead of spin 3/2.

Observing the spin of these baryons
thus proves the existence of color.



Y
Δ^-
Δ^0
Δ^+
Δ^{++}
1
1232MeV
Σ^{-*}
Σ^{0*}
Σ^{+*}
I_3
1385MeV
-1
-1/2
1/2
1
Ξ^{-*}
Ξ^{0*}
1530MeV
−1
Ω^-
1672MeV

Spin 3/2

Spin 1/2