Physics 214 UCSD

Lecture 4

- Symmetries & QCD
  - Greiner QM part 2: Symmetries
  - Halzen & Martin
    - "Low energy" QCD is close to impossible to calculate. Accordingly, symmetries, even approximate ones, play and important role in QCD, *e.g.*:

    ⇒ Meson & Baryon spectroscopy

    ⇒ Hadronic decays

    ⇒ Pion-nucleon scattering

    ⇒ …

Disclaimer

- Don’t expect mathematical rigor !!!
- E.g.:
  - I call semi-simple Lie groups simply Lie groups.
  - If you find other simplifications that I gloss over, feel free to point them out.

Origin of Symmetry in QCD

- QCD is flavor blind, *i.e.*, independent of the quark flavor.
- Flavor symmetry of interactions as long as quark masses can be considered the same.
  - $m_u = m_d \Rightarrow$ SU(2) symmetry called "Isospin".
  - $m_s = m_u = m_d \Rightarrow$ SU(3) flavor symmetry of u,d,s
  - $m_c , m_b \approx \infty \Rightarrow$ HQET
- As $\alpha_s \sim 1$ for energies relevant to the physics of hadrons, perturbation theory is not useful.

  $\Rightarrow$ Symmetries play an important role in physics of hadrons!

Symmetries in QM

- a reminder -

- Time evolution of a state ψ is fully described by:

$$ih\frac{\partial}{\partial t}\psi = H\psi$$

- It is obviously of great practical value to understand the complete set of symmetry operations S: $S\,\psi = \psi'$ for which ψ' has the same time evolution as ψ.
- This implies: $S\,H\,S^{-1} = H$ or $[H,S]=0$

Being Pedantic

$$ih\frac{\partial}{\partial t}\psi = H\psi$$

$$S\psi = \psi'$$

$$ih\frac{\partial}{\partial t}\psi' = H\psi'$$

$$ih\frac{\partial}{\partial t}S\psi = HS\psi$$

$$ih\frac{\partial}{\partial t}S^{-1}S\psi = S^{-1}HS\psi$$

$$\Rightarrow S^{-1}HS = H$$

$$\Rightarrow HS = SH \Rightarrow [H, S] = 0$$

If you know the symmetry for a hamiltonian you obviously save yourself a lot of unnecessary calculations because given a solution, you know how to produce other solutions, without solving the Schroedinger Equation for each.

Moreover, you can relate two different solutions, neither of which you might be able to calculate, to each other, and thus estimate ratios of processes.

The latter is an invaluable tool for low energy QCD, because perturbation theory fails there.

You may also be able to predict that certain processes are forbidden because they would violate a symmetry of QCD.
You will exercise this in this week's homework!

Group Theory

The set $G = \{S_1,\ldots,S_n\}$ is called a group if:

1. A product "x" is defined such that
   $S_m \times S_k = S_l$ with $S_l \in G \;\; \forall\, m,k$
   (it "closes" under multiplication)
2. An element $S_0 \in G$ exists for which
   $S_0 \times S_k = S_k \quad \forall\, k$
   (existence of the "identity")
3. $\forall\, m$ there is a k such that $S_m \times S_k = S_0$
   (for each element there is an "inverse")
4. Multiplication is associative

Utility of Group Theory

The set $G = \{S_1,\ldots,S_n\}$ of physical transformations of a system (*e.g.*, time evolution, rotations…) with multiplication defined as composition (one transformation followed by another) is a group:

1. A product "x" is defined such that
   S_m x $S_k = S_l$ with $S_l \in G \;\; \forall\, m,k$
   ($\psi_i \rightarrow \psi_j \rightarrow \psi_k$ then also $\psi_i \rightarrow \psi_k$)
2. An element $S_0 \in G$ exists for which
   S_0 x $S_k = S_k \quad \forall$ k
   (identity: $\psi_i \rightarrow \psi_i$)
3. $\forall\, m$ there is a k such that S_m x $S_k = S_0$
   (inverse: $\psi_i \rightarrow \psi_k$ then also $\psi_k \rightarrow \psi_i$)
4. Multiplication is associative
   $(\psi_i \rightarrow (\psi_j \rightarrow \psi_k) = (\psi_i \rightarrow \psi_j) \rightarrow \psi_k)$

Lie groups are especially useful

- *G* is a group for which all elements are infinitely differentiable functions of some set of parameters. E.g.:

$$S(\alpha_i,\ldots,\alpha_n) = e^{-i\sum_{j=1}^{n}\alpha_j L_j}$$

$$\left.\frac{\partial S}{\partial \alpha_j}\right|_{\alpha_j = 0} = -iL_j$$

- L_j are called the "generators" of the group *G*.

  Sort of like basis vectors to span *S*.

- It is obviously useful to know the complete Lie group *G* of *H* because it allows straightforward construction of all states ψ' that have the same time evolution as ψ.

- In the following, we will go through some characteristics of Lie groups without proof.

- For more details, see Greiner chapters 1-4.

- Generators of G form an orthogonal set.
  (Need inner product. E.g., if L_m are hermitian matrices one can use the trace to define $\langle L_m , L_n \rangle = \mathrm{Tr}(L_m L_n)$)

- Lie group is fully characterized by the commutator relationship among its generators:

  $[L_k, L_l] = ic_{klm} L_m$

  This equation is thus called the "Lie algebra" of G and c_{klm} are the "structure constants" of the Lie Algebra.
  (More precisely the Lie algebra of G is the vector space spanned by the L_m's, together with the rule that it closes under commutation)

- The number of L_m's is the "dimension" of the Lie algebra

- The rank k of G is the maximum number of mutually commuting generators
  (Why do we care?: The set UL_mU^{-1} satisfies the same commutation relation. Choose U to diagonalize k generators. The diagonal set is called the "Cartan subalgebra")

- Theorem of Racah:

  For every Lie group G of rank k there is a set of exactly k "Casimir" operators $C_1,\ldots,C_k$ that commute with every operator in G, including themselves.
  ("operators" because they act on states, just like the group elements)

- A hamiltonian H that has the symmetry G will have exactly $2k$ good quantum numbers, in addition to E.
  (i.e., k Casimir eigenvalues and k eignevalues of the Cartan subalgebra)

- It can be shown that any operator A that commutes with all operators in G (*i.e.* commutes with the generators) must be a function of the Casimir operators. This implies that $E = E(C_1,\ldots,C_k)$.

Importance to Physics

- The Hilbert Space of all states ψ that satisfy the Schroedinger equation is divided into “multiplets” characterized by the value for the set of *k* Casimir operator eigenvalues.
- Transitions between multiplets do not happen.
- All states within a given multiplet have the same energy.
- Out of the *N* generators of the Lie group, a set of *k* (generally $k < N$ except for abelian groups for which $k = N$) can be diagonalized with H simultaneously, thus providing the second set of *k* good quantum numbers, in addition to the *k* Casimir operators.

Summary

- Let G be the N dimensional Lie group of Rank k for the Hamiltonian H.
- Then we have the following set of operators that mutually commute:

$$H, C_1,\ldots,C_k, L_1,\ldots,L_k$$

- Any state is thus characterized by $2k$ quantum numbers.
- The energy E is given as some function of the $C_1,\ldots,C_k$.

Examples

- Translation Group
- Rotation Group
- Flavor SU(3)

Translation group

- Translations commute with each other.
- The generators of the translation group thus commute.
- All generators are thus Casimir operators of the group.
- The generators of the group are the momentum operators p_x, p_y, p_z

Group of Rotations in 3-space

- Generators: J_x, J_y, J_z
- Lie algebra: $[J_k, J_l] = i\,\varepsilon_{klm}\,J_m$
- Rank = 1
- Casimir Operator: J^2
- Multiplets are classified by their total angular momentum J
- States are classified by J and J_z, the latter being one of the three generators.

Group Representations

- A set of N x N matrices is called an N-dimensional representation of a Lie group if there is a one-to-one map: $L_k \leftrightarrow M_k$ such that $[M_k, M_l] = ic_{klm} M_m$ with c_{klm} being the structure constants of the Lie group.

Back to rotation group

- SO(3) = group of orthogonal 3×3 matrices with determinant = 1
- SU(2) = group of 2×2 unitary matrices with determinant = 1
- Structure constants: ϵ_{ijk}
- Aside: SU(2) is sometimes used as name for the more general rotation group, not just the 2×2 unitary traceless matrices. I will do that from now on because it's shorter than writing "rotation group".

SU(2) and Spin

- Spin 1/2 is the fundamental representation of spin: all other spin states can be constructed by angular momentum addition of spin 1/2

$\Rightarrow$ Fundamental representation of the rotation group.

$\Rightarrow$ Fundamental representation of “SU(2)”

Reminder: Generators of SU(2)

$$K_i = \frac{1}{2}\sigma_i$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \sigma_x$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \sigma_y$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \sigma_z$$

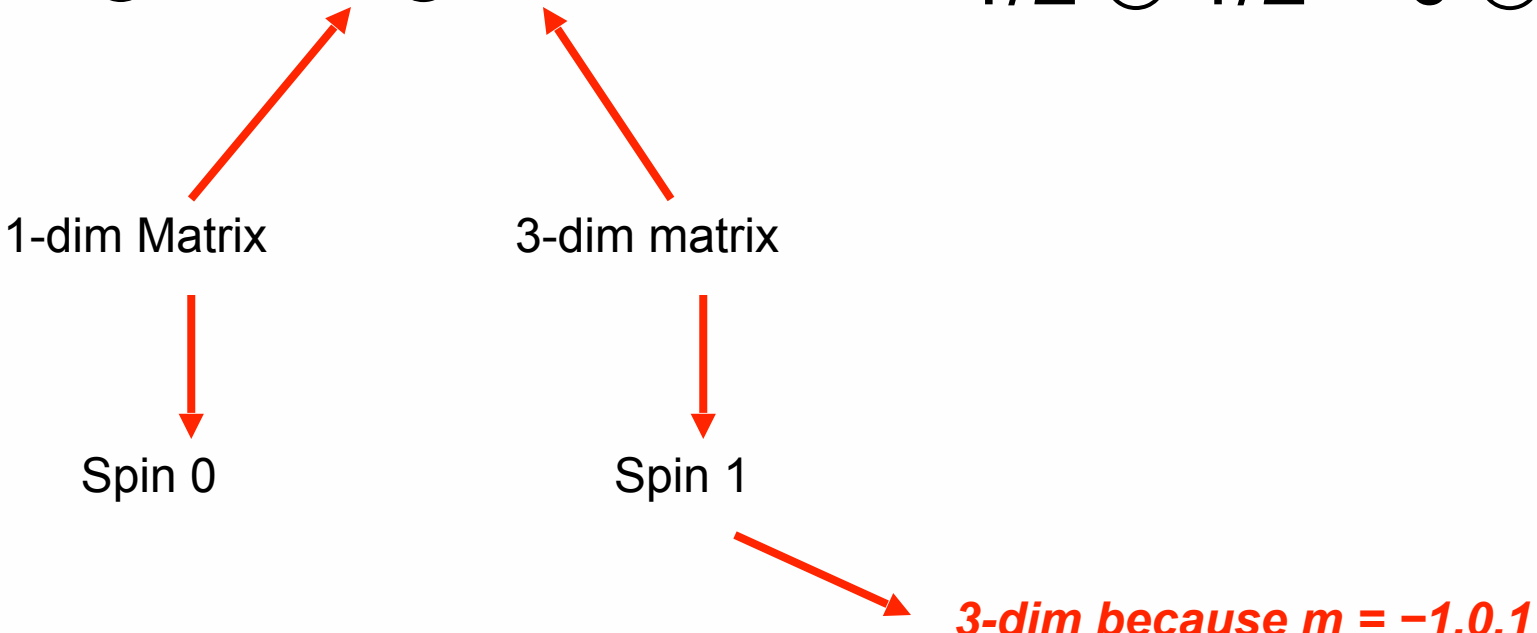
Arbitrary rotation in spin space:

$$\begin{pmatrix} |u'\rangle \\ |d'\rangle \end{pmatrix} = e^{i\sum_{k=1}^{3}\alpha_k\sigma_k} \begin{pmatrix} |u\rangle \\ |d\rangle \end{pmatrix}$$

And we combine spins the same way we have always combined them, using Clebsch-Gordan coefficients.

Language Comparison

- Group theory:
  $2 \otimes 2 = 1 \oplus 3$
- Spin in QM:
  $1/2 \otimes 1/2 = 0 \oplus 1$



Reducible vs Irreducible Representation

$$\begin{pmatrix} (1 \times 1) & 0 \\ 0 & \begin{pmatrix} 3 & x & 3 \end{pmatrix} \end{pmatrix}$$

This describes:

$$2 \otimes 2 = 1 \oplus 3$$

Or in spin language: This 4×4 matrix describes a system with spin 0 and spin 1 as irreducible subspaces.

Spin 3/2

$$S_z = \frac{1}{2}\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

This describes the z-component of a spin in the sense that:

$$|j = 3/2; m = 3/2\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The 4x4 matrices are thus used here to describe the dim-4 multiplet of a spin 3/2 particle.
Arbitrary rotations in this space are thus implemented via the generators of the 4×4 representation of the rotation group.

$1 \oplus 3$ vs spin 3/2

- Both are described by 4×4 matrices.
- Both are representations of SU(2), the rotation group.
  - Both are described by two eigenvalues
  - Both have generators that have the same commutator relationship as SU(2).
- However the matrices for $1 \oplus 3$ are a reducible representation while the matrices for spin 3/2 are an irreducible representation of SU(2).

Commonality of all representations of the rotation group.

- Rank = 1

$\Rightarrow$ there are only two good quantum numbers !!!

$\Rightarrow |J;m\rangle$ the eigenvalues of J^2 and J_z

What if the physics has more conserved quantum numbers?

Then there must be a higher rank symmetry group that describes the physical system !!!

Examples of higher rank symmetry groups

$SU(n) \Rightarrow \text{Rank} = n - 1$

$\Rightarrow$ # of generators = $n^2 - 1$

SU(n) = group of $n \times n$ unitary matrices with determinant = 1

Applications of Flavor SU(2)

- Spectroscopy
- Scattering
- Partial decay widths

Spectroscopy of Isospin = 1/2

- Quarks:

$$|u\rangle = \left|\frac{1}{2}; T_3 = +\frac{1}{2}\right\rangle \qquad m_u = 1.5 \text{ to } 3 \text{ MeV}$$

$$|d\rangle = \left|\frac{1}{2}; T_3 = -\frac{1}{2}\right\rangle \qquad m_d = 3 \text{ to } 7 \text{ MeV}$$

- Mesons:

$$|K^{(*)+}\rangle = \left|\frac{1}{2}; T_3 = +\frac{1}{2}\right\rangle \qquad m = 493\ (892)\text{MeV}$$

$$|K^{(*)0}\rangle = \left|\frac{1}{2}; T_3 = -\frac{1}{2}\right\rangle \qquad m = 497\ (896)\text{MeV}$$

- Baryons:

$$|p\rangle = \left|\frac{1}{2}; T_3 = +\frac{1}{2}\right\rangle \qquad m = 938.2 \text{ MeV}$$

$$|n\rangle = \left|\frac{1}{2}; T_3 = -\frac{1}{2}\right\rangle \qquad m = 939.5 \text{ MeV}$$

Aside on K^* Meson

- With regard to flavor SU(2), this is a doublet.
  - Isospin = $+1/2, -1/2$
- With regard to spin SU(2), this is a triplet.
  - Spin = $+1, 0, -1$

Spectroscopy of Isospin = 1

- $2 \otimes 2 = 1 \oplus 3 \Rightarrow$ a singlet and a triplet.
- Pseudoscalar Mesons:

$$\left|\pi^{+}\right\rangle = \left|1;+1\right\rangle$$ m = 139 MeV

$$\left|\pi^{0}\right\rangle = \left|1;0\right\rangle$$ m = 135 MeV

$$\left|\pi^{-}\right\rangle = \left|1;-1\right\rangle$$ m = 139 MeV

$$\left|\eta_{0}\right\rangle = \left|0;0\right\rangle$$

Same repeats for vector mesons
ρ^{+} ρ^{0} ρ^{-} and ω
… and so forth …

Language comparison

- $2 \otimes 2 = 1 \oplus 3 \Rightarrow$ a singlet and a triplet.
- Spin 1/2 + Spin 1/2 $\Rightarrow$ spin 0 and spin 1

The fact that we are talking SU(2) flavor instead of SU(2) spin makes no difference to the formalism.

Spin and flavor are just two physical effects that are both described by the same math: Group Theory.

Isospin and Scattering

- Decompose the initial state into its spin states.
- Decompose the final state into its spin states.
- Recall Wigner-Eckart Theorem

$$\langle jm|T^k_q|j'm'\rangle = \langle j||T^k||j'\rangle C^{jm}_{kqj'm'}$$

- Recall that QCD is isospin conserving, because it is flavor blind.

Scattering

$$\frac{\sigma(pp \to \pi^+ d)}{\sigma(np \to \pi^0 d)} = \frac{\left|\langle \pi^+ d | S | pp \rangle\right|^2}{\left|\langle \pi^0 d | S | np \rangle\right|^2} \bullet PhSpR = \frac{\left|\langle 1 || S || 1 \rangle\right|^2}{\left|\langle 1 || S || 1 \rangle + \langle 1 || S || 0 \rangle\right|^2 \bullet \frac{1}{2}} = 2$$

$$|pp\rangle = \left|\frac{1}{2};+\frac{1}{2}\right\rangle \left|\frac{1}{2};+\frac{1}{2}\right\rangle = |1;1\rangle_{NN}$$

$$|np\rangle = \left|\frac{1}{2};-\frac{1}{2}\right\rangle \left|\frac{1}{2};+\frac{1}{2}\right\rangle = \frac{1}{\sqrt{2}}\left(|1;0\rangle_{NN} + |0;0\rangle_{NN}\right)$$

$$|\pi^+ d\rangle = |1;1\rangle_\pi |0;0\rangle_d = |1;1\rangle_{\pi d}$$

$$|\pi^0 d\rangle = |1;0\rangle_\pi |0;0\rangle_d = |1;0\rangle_{\pi d}$$

No Isospin = 0 component

$\langle 1 || S || 0 \rangle = 0$ ☜ QCD is flavor blind

Aside on Triangle Relationships

- Often, there are more than one reduced matrix elements contributing to a process.
- Sometimes one can relate processes where one contributes with processes where two contribute (that is, isospin gives a + b + c = 0)
- This leads to triangle relationships
  (three complex numbers that sum to zero form a triangle in the *z*-plane)
- See homework for an example.

Partial Decay Width

- K*(892) decays ~100% to Kπ
- The partial decay width is so well determined by isospin that the PDG doesn't even bother writing it down.

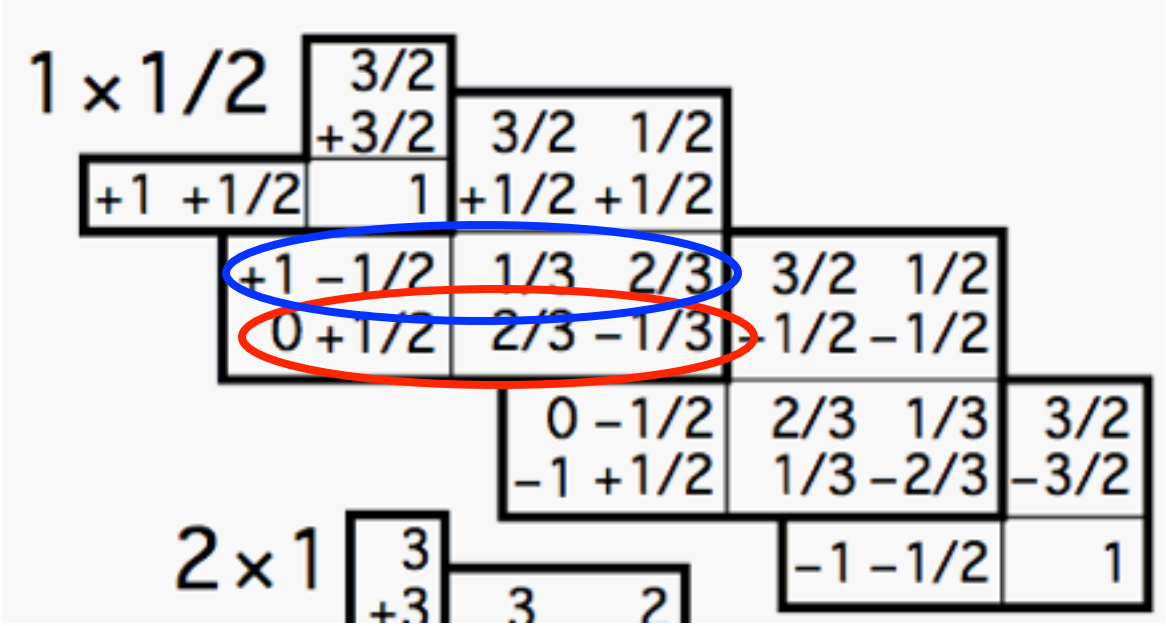
$$\frac{\Gamma(K^{*+} \to K^{+}\pi^{0})}{\Gamma(K^{*+} \to K^{0}\pi^{+})} = \frac{1/3}{2/3} = 1/2$$

$$\begin{pmatrix} K^{(*)+} = u\bar{s} \\ K^{(*)0} = d\bar{s} \end{pmatrix} = \begin{pmatrix} +\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$$

$$\left|K^{+}\pi^{0}\right\rangle = \left|\frac{1}{2};+\frac{1}{2}\right\rangle_{K} \left|1;0\right\rangle_{\pi} = \sqrt{\frac{2}{3}}\left|\frac{3}{2};+\frac{1}{2}\right\rangle_{K\pi} - \sqrt{\frac{1}{3}}\left|\frac{1}{2};+\frac{1}{2}\right\rangle_{K\pi}$$

$$\left|K^{0}\pi^{+}\right\rangle = \left|\frac{1}{2};-\frac{1}{2}\right\rangle_{K} \left|1;1\right\rangle_{\pi} = \sqrt{\frac{1}{3}}\left|\frac{3}{2};+\frac{1}{2}\right\rangle_{K\pi} + \sqrt{\frac{2}{3}}\left|\frac{1}{2};+\frac{1}{2}\right\rangle_{K\pi}$$

Clebsh-Gordon Reminder



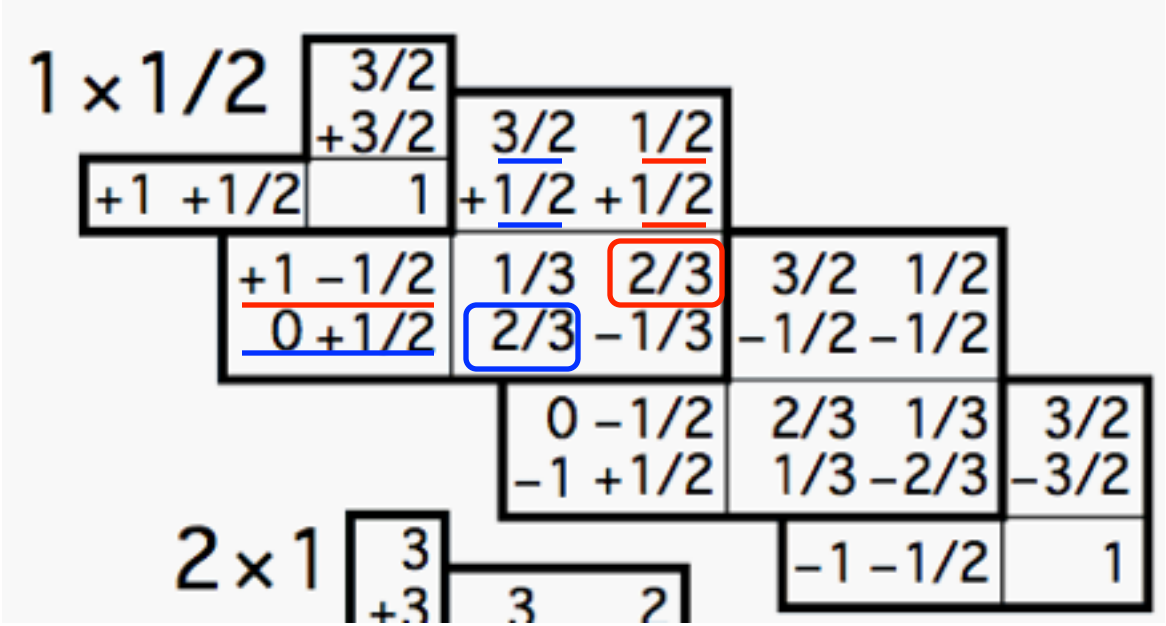
π^0 K^+

(1;0) + (1/2;+1/2) = sqrt(2/3) (3/2;1/2) - sqrt(1/3) (1/2;1/2)

(1;1) + (1/2;-1/2) = sqrt(1/3) (3/2;1/2) + sqrt(2/3) (1/2;1/2)

π^+ K^0

Clebsh-Gordon Reminder



π^0 K^+

(1;0) + (1/2;+1/2) = sqrt(2/3) (3/2;1/2) - sqrt(1/3) (1/2;1/2)

(1;1) + (1/2;-1/2) = sqrt(1/3) (3/2;1/2) + sqrt(2/3) (1/2;1/2)

π^+ K^0

Isospin for anti-quarks

- We want charge to be conserved.
  - This requires putting the most positively charged state always at the top in our isospin doublets.
- We want to distinguish quark and antiquark formally.
  - Add a “bar” on top of the letter.

$$\begin{pmatrix} u \\ d \end{pmatrix} \xrightarrow{?} \begin{pmatrix} \bar{d} \\ \bar{u} \end{pmatrix}$$

- Want to make anti-quark doublet with same transformation properties as quarks
  - This will allow us to have the same clebsh-gordon coefficients for both, thus making it possible to combine quarks and antiquarks without thinking too much.

Isospin for anti-quarks

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \sigma_y$$

- Want to make anti-quark doublet with same transformation properties as quarks

$$\begin{pmatrix} u \\ d \end{pmatrix} \longrightarrow \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix}$$

We need an extra minus sign in the definition, in addition to "bar and flip".

$$\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow e^{\frac{1}{2} i\theta_y \sigma_y} \begin{pmatrix} u \\ d \end{pmatrix} = \left[\cos\frac{\theta_y}{2} + i \sin\frac{\theta_y}{2} \sigma_y \right] \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta_y}{2} & \sin\frac{\theta_y}{2} \\ -\sin\frac{\theta_y}{2} & \cos\frac{\theta_y}{2} \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}$$

If you simply bar and flip then you get the wrong sign
In front of the "sin" terms.

$$\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow e^{\frac{1}{2} i\theta_y \sigma_y} \begin{pmatrix} u \\ d \end{pmatrix} = \left[\cos\frac{\theta_y}{2} + i \sin\frac{\theta_y}{2} \sigma_y \right] \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta_y}{2} & \sin\frac{\theta_y}{2} \\ -\sin\frac{\theta_y}{2} & \cos\frac{\theta_y}{2} \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}$$

$$\begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} \rightarrow e^{\frac{1}{2} i\theta_y \sigma_y} \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} = \left[\cos\frac{\theta_y}{2} + i \sin\frac{\theta_y}{2} \sigma_y \right] \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta_y}{2} & \sin\frac{\theta_y}{2} \\ -\sin\frac{\theta_y}{2} & \cos\frac{\theta_y}{2} \end{pmatrix} \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix}$$

Note:
The point here is that you want to be able to derive the rotated doublet either via rotation to the quark doublet followed by Charge conjugation and flip, or by starting with the anti-*q* doublet and using the same rotation as for *q* doublet.

Quantum Numbers for Mesons

- J^{PC}

  J = total angular momentum = L + S

  P = parity

  C = charge conjugation

- Only neutral particles can be eigenstates of C, of course.

Generalized Pauli Principle

- The fermion-antifermion wave function must be odd under interchange of all coordinates (space, spin, charge).
  - Space interchange $\rightarrow (-1)^{L}$
  - Spin interchange $\rightarrow (-1)^{S+1}$
  - Particle-antiparticle interchange $\rightarrow$ eigenvalue of C
- Bottom line (neutral mesons):

$$(-1)^{L+S+1}\, C = -1 \quad \Rightarrow \quad C = (-1)^{L+S}\,; \;\; P = (-1)^{L+1}$$

π^0 : $C = (-1)^{0+0} = 1$; $P = (-1)^{0+1} = -1$ $\Rightarrow$ pseudoscalar meson

ρ^0 : $C = (-1)^{0+1} = -1$; $P = (-1)^{0+1} = -1$ $\Rightarrow$ vector meson

b_1 : $C = (-1)^{1+0} = -1$; $P = (-1)^{1+1} = +1$ $\Rightarrow$ axial vector meson

You will use this in the homework.

What's the J^{PC} of the initial state?
What J^{PC} can I construct from the final state particles?
J^{PC} is conserved on QCD, and thus not all final state combinations may be allowed, in general.

Example: π Wave Function

We should get that $C = +1$ for π^0 because $\pi^0 \to 2\gamma$ (photons).
Photons are $C = -1$ because they are produced by moving charges which have $C = -1$, of course. Accordingly, $\pi^0 \to 3\gamma$ is heavily suppressed.

$$|I = 1; I_3 = 0\rangle = \tfrac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

$$|S = 0\rangle = \tfrac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$$

$$|color\rangle = \tfrac{1}{\sqrt{3}}(R\bar{R} + B\bar{B} + G\bar{G})$$

$$|\pi^0\rangle = \frac{1}{\sqrt{12}} \sum_{a=R,G,B} |u_a\uparrow\ \bar{u}_{\bar{a}}\downarrow\rangle - |u_a\downarrow\ \bar{u}_{\bar{a}}\uparrow\rangle + |d_a\uparrow\ \bar{d}_{\bar{a}}\downarrow\rangle - |d_a\downarrow\ \bar{d}_{\bar{a}}\uparrow\rangle$$

Verify $C = +1$:

$$C|\pi^0\rangle = \frac{1}{\sqrt{12}} \sum_{a=R,G,B} |\bar{u}_{\bar{a}}\uparrow\ u_a\downarrow\rangle - |\bar{u}_{\bar{a}}\downarrow\ \ u_a\uparrow\rangle + |\bar{d}_{\bar{a}}\uparrow\ d_a\downarrow\rangle - |\bar{d}_{\bar{a}}\downarrow\ d_a\uparrow\rangle$$

$$= \frac{1}{\sqrt{12}} \sum_{a=R,G,B} -|u_a\downarrow\ \bar{u}_{\bar{a}}\uparrow\rangle + |\ u_a\uparrow\ \bar{u}_{\bar{a}}\downarrow\rangle - |d_a\downarrow\ \bar{d}_{\bar{a}}\uparrow\rangle + |d_a\uparrow\ \bar{d}_{\bar{a}}\downarrow\rangle$$

SU(3) Next Lecture