Physics 214 UCSD
Experimental Particle Physics

Lecture 3

Fast forward through HEP

Range of force for massive mediator

- We have two ways of handwaving our way to see finite range:
  - Uncertainty principle of Energy and time
  - Yukawa potential as solution to Klein-Gordon equation

Range of "force" as quantum fluctuation

$$\Delta E \Delta t \approx h$$

$$\Delta E = mc^2$$

$$\Rightarrow \Delta t \approx \frac{h}{mc^2}$$

$$R \approx c\Delta t = \frac{h}{mc}$$

$$\mathbf{R \propto 1/m}$$

Range of force is inverse proportional to mass of mediator.

A bit more rigorous: Yukawa

Static source of "charge"
=> Spherical potential.

$$U(r) = \frac{Q}{r} e^{-r/R}$$

As solution to:

$$E^2 = P^2 + m^2 \Leftrightarrow -\nabla^2 \Phi + m^2 \Phi = 0$$

Given that QM tells us:

$$\vec{P} = -i\vec{\nabla}$$

$$E = i\partial / \partial t \Rightarrow 0$$ due to static potential

We then get:

$$\nabla^2 \Phi = m^2 \Phi = \frac{\Phi}{R^2}$$

Yukawa Potential

$$\nabla^2 \Phi = m^2 \Phi = \frac{\Phi}{R^2}$$

- For $r \neq 0$ this equation is solved by:

$$\Phi(r) = \frac{Q}{r} e^{-r/R}$$

A potential with a characteristic range R,
and a "charge" or "coupling strength" Q.

Add to this Fermi's Golden Rule:

- Incoming plane wave => outgoing whatever
- Rate of transition = $2\pi\, |M_{if}|^2\, \rho(E_f)$
- With: $M_{if} = \int \psi_f^*\, U(r)\, \psi_i\, \mathrm{dVol}$
- As the wave functions are plane waves, this is nothing more than the fourier transform of the potential, with *k* being the momentum transfer in the collision.

For Yukawa:

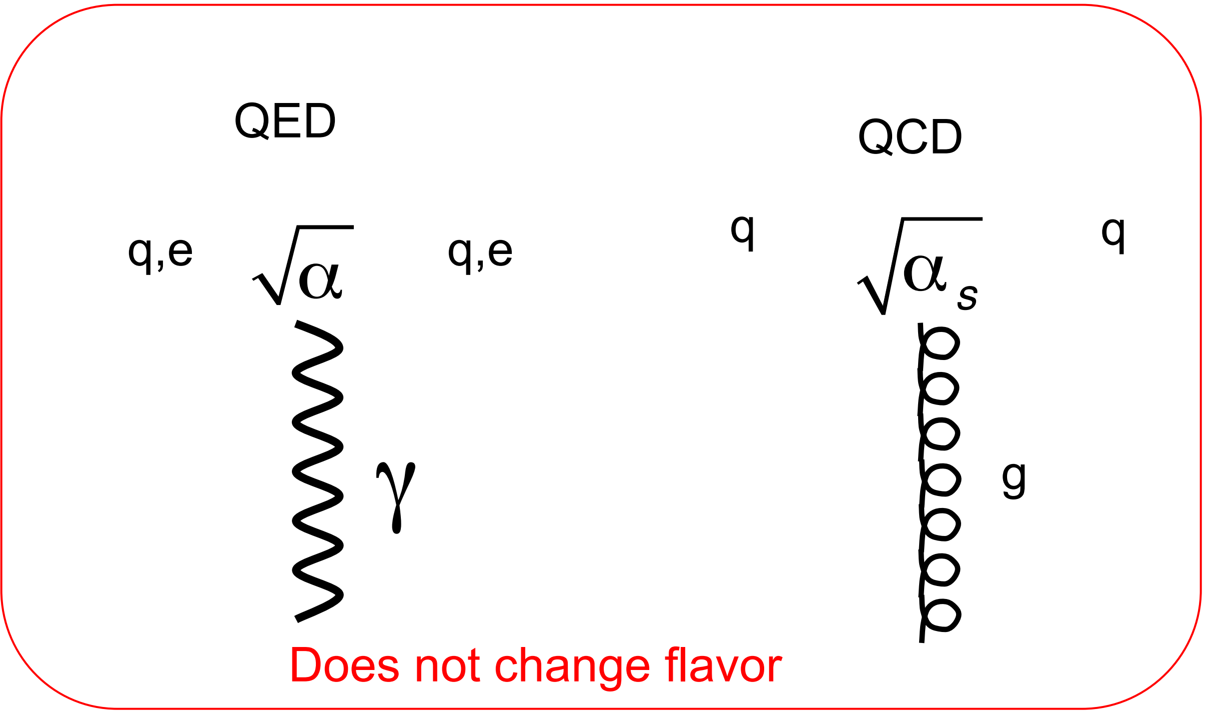
$$M_{if}(k^2) \propto qQ \bullet \frac{1}{k^2 + m^2}$$

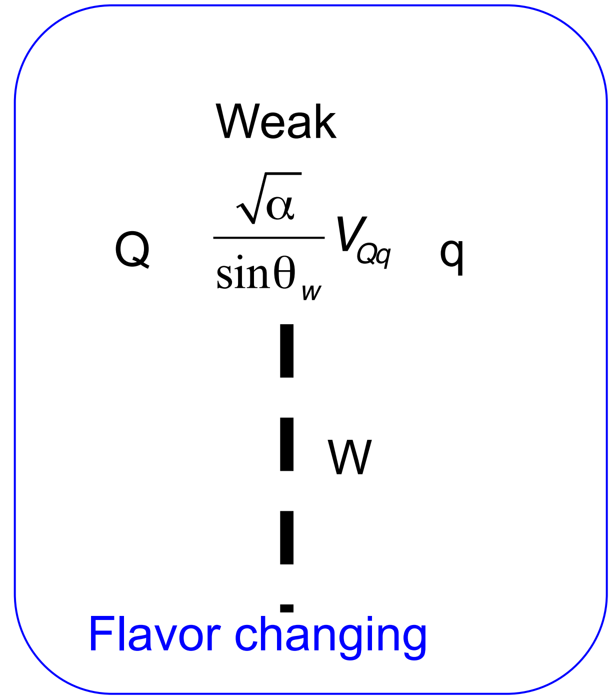
Things to remember:

- Rate of transition $\propto$ $|\text{Amplitude}|^2$
- Amplitude = vertex factors * propagator

All of this is for single boson exchange,
i.e. leading order process only!

Rules for Standard Model Interactions





Amplitude = vertex factors times propagators

Note: The formalism is the same with new physics. All you do is add new particles and rules for the interactions.

Orders of magnitude of interactions:

Interaction	Typical τ	Typical σ	coupling
strong	$\sim 10^{-23}$ sec $\Delta \to p\pi$	~ 10mb $p\pi \to p\pi$	$\alpha_s \sim 1$
EM	$\sim 10^{-16..-20}$ s $\pi^0 \to \gamma\gamma$	$\sim 10^{-3}$mb $\gamma p \to p\pi^0$	$\alpha_{EM} \sim 10^{-2}$
weak	$> \sim 10^{-12}$ sec $\pi^- \to \mu^- \nu$	$\sim 10^{-11}$mb $\nu p \to e^- p\pi^+$	α_{EM} with massive propagator

Can we understand these numbers?

Note: 1 barn = 10^{-28} m^2

1st order = coupling^2 x propagator^2

EM & Strong mediated by massless particles …
… but with different couplings.

$$\frac{\alpha_s}{\alpha_{EM}} \sim 10^2$$

EM & weak have ~ same coupling …
… but with different mass for propagator.

Processes we listed have roughly k~1GeV

$$\frac{EM}{Weak} \sim \left(\frac{\frac{1}{k^2}}{\frac{1}{k^2 + m_W{}^2}} \right)^2 = \frac{m_W{}^4}{k^4} \sim 10^8$$

Impressive how well these simple relative estimates work!

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10^4

10^8

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What about estimating the absolute scale?

Assume pion-proton scattering is nothing more than
Solid sphere's hitting each other:

$$\sigma \sim A \sim \pi R^2 \sim 3\ (1\text{fm})^2 \sim 30\text{mb}$$

Lifetime of strong decaying particle is defined by range based
on exchange of lightest colorless hadron:

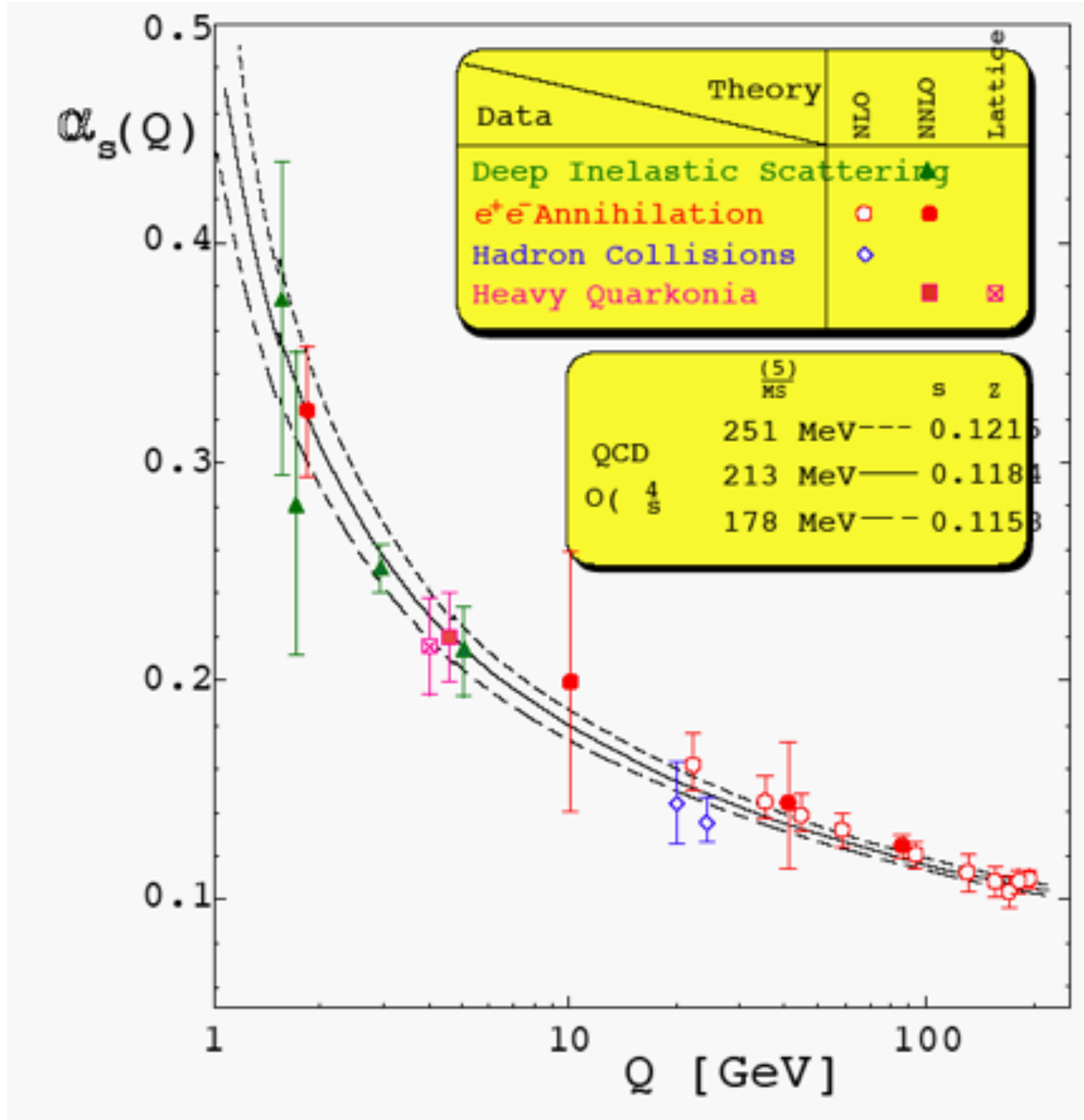
$$\tau \sim 1/m_\pi \sim 1/100\text{MeV} \sim 10^{-23}\ \text{sec}$$

Sort of works in both cases.

Aside on running couplings

Couplings depend on momentum transfer, Q

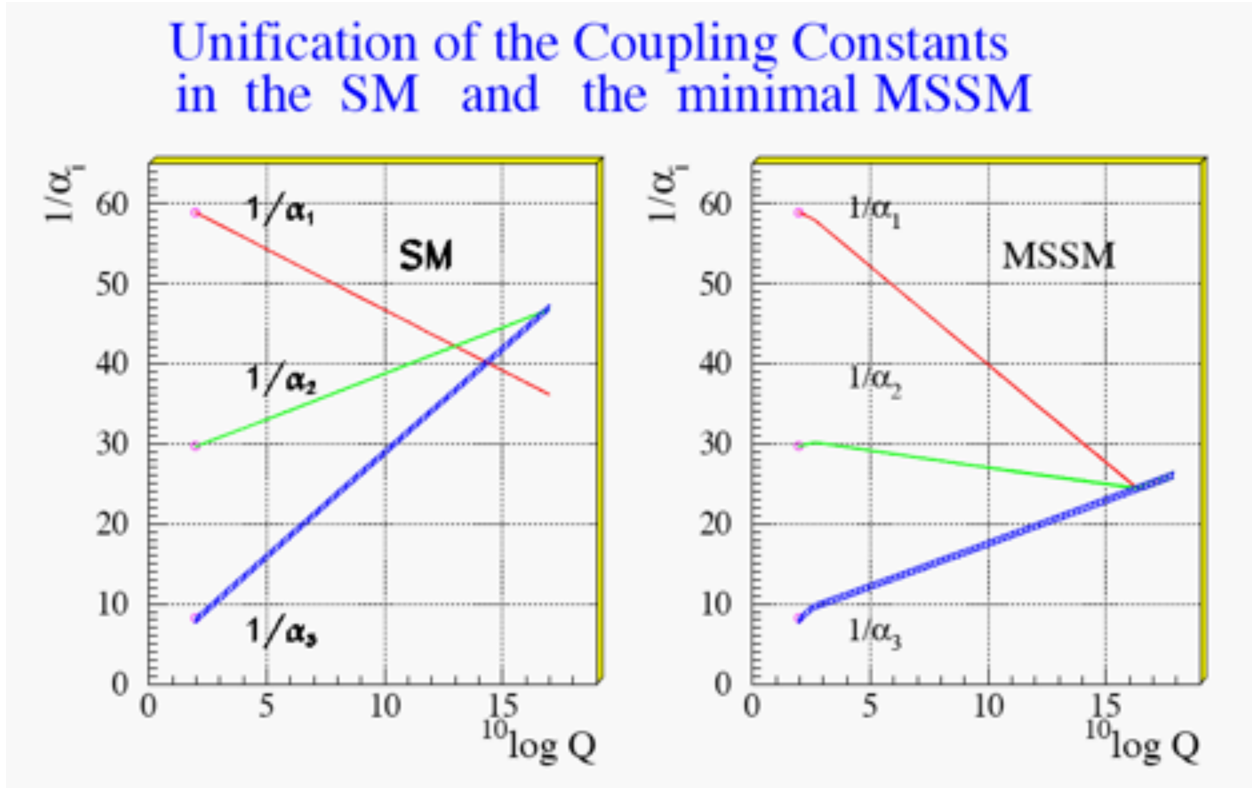
Strong coupling is O(1) at the scale of hadron masses, thus ***confinement***, but becomes O(0.1), and thus perturbative, at O(100GeV), i.e. ***asymptotic freedom.***



hep-ph/0012288v2

Coupling Unification ???

The "Q" here is actually k^2/μ^2, with μ being a reference scale, e.g. M_Z , at which the couplings are measured.



Details of the running depends on gauge boson self-couplings, # of families, and # of Higgs doublets, and Particle content and Masses in the theory.

$$\begin{aligned}\alpha_1 &= (5/3)g'^2/(4\pi) = 5\alpha/(3\cos^2\theta_W),\\ \alpha_2 &= g^2/(4\pi) = \alpha/\sin^2\theta_W,\\ \alpha_3 &= g_s^2/(4\pi)\end{aligned}$$

hep-ph/0012288v2

Aside on lifetime of unstable particles

Issues around unstable particles (1)

- Assume we have a large number N of particles of a certain type, at $t = t_0$. How many are left at $t = t_0 + dt$?

$p(t)dt$ = prob. for decay during dt $= k\, dt$

$P(t)$ = prob. for survival at t

Exponential decay law follows directly from assumption of constant rate of decay, i.e. transition rate that is independent of N(t=0).

$$P(t+dt) = P(t)(1-kdt)$$

$$-kP(t) = \frac{dP}{dt}$$

$$P(t) = N(t=0) \bullet e^{-kt}$$

$$\tau \equiv 1/k \Rightarrow P(t) = N(t=0) \bullet e^{-t/\tau}$$

Issues around unstable particles (2)

- We refer to τ as the "lifetime" of the particle because $< t >_{\text{decay}} = \tau$
- We refer to $\Gamma = 1/\tau$ as the Total Width, or total decay rate.
- In general, a particle may decay via more than one path, or into more than one distinct final state, *e.g.*, $Z \to e^+e^-$, $\mu^+\mu^-$, etc. We refer to the decay rate into a given final state as the partial with, Γ_i .
  - The total width is given by the sum of all partial widths.
- We refer to the ratio of Γ_i / Γ as the "branching ratio" into the final state i.
  - The sum of all branching ratios adds up to 1.

Citation: K.A. Olive *et al.* (Particle Data Group), Chin. Phys. **C38**, 090001 (2014) (URL: http://pdg.lbl.gov)

Δ BARYONS ($S=0$, $I=3/2$)

$\Delta^{++} = uuu$, $\Delta^{+} = uud$, $\Delta^{0} = udd$, $\Delta^{-} = ddd$

$\Delta(1232)\ 3/2^+$

$I(J^P) = \frac{3}{2}(\frac{3}{2}^+)$

Breit-Wigner mass (mixed charges) = 1230 to 1234 ($\approx$ 1232) MeV

Breit-Wigner full width (mixed charges) = 114 to 120 ($\approx$ 117) MeV

$p_{\text{beam}} = 0.30$ GeV/c $\qquad 4\pi\lambda\!\!\!^{-2} = 94.8$ mb

Re(pole position) = 1209 to 1211 ($\approx$ 1210) MeV

-2Im(pole position) = 98 to 102 ($\approx$ 100) MeV

Δ(1232) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\pi$	100 %	229
$N\gamma$	0.55–0.65 %	259
$N\gamma$, helicity=1/2	0.11–0.13 %	259
$N\gamma$, helicity=3/2	0.44–0.52 %	259

Nγ is electromagnetic, *Nπ* is strong, ratio is… α_{em}/α_s

Issues around unstable particles (3)

- *What's the mass of an unstable particle?*

  $\Delta E \, \Delta t \sim 1$

  In rest frame $E = M$,

  In general $\Delta t \sim \tau$, $\Rightarrow$ $\boldsymbol{\Delta M \sim \Gamma}$

- *If mass isn't well defined, then what's the probability distribution for finding a particle with a given mass?*
  - We call this the "lineshape" of the particle.

$$|\psi(t)\rangle = |\psi(0)\rangle e^{-iE_0 t}$$

$$\langle\psi(t)|\psi(t)\rangle = 1$$ ⟵ Normalization for stable particle.

$$\langle\psi(t)|\psi(t)\rangle = e^{-t/\tau}$$ ⟵ Normalization for unstable particle.

We can get to this normalization if we replace:
E_0 by $E_0 - i\,\Gamma/2$.

We then get the lineshape from fourier transformation:

$$|\psi(E)\rangle = \frac{1}{\sqrt{2\pi}}\int_0^\infty dt e^{iEt}|\psi(t)\rangle = \frac{1}{\sqrt{2\pi}}\int_0^\infty dt e^{i(E-E_0-\Gamma/2)t}|\psi(0)\rangle$$

$$\langle\psi(E)|\psi(E)\rangle = \frac{1/2\pi}{(E-E_0)^2+(\Gamma/2)^2}$$

Identify E_0 with M and you get the non-relativistic Breit-Wigner lineshape.

In real live, hadronic resonances are not this simple because …

- Interference with higher resonances.
- Total width "depends" on *E*.
- Phase space affects lineshape
- Finite size effects (Blatt-Weisskopf barrier penetration factor)

I'll show you examples for the first 3.

http://www.t2.ucsd.edu/twiki2/pub/UCSDTier2/Physics214Spring2015/ajw-breit-wigner-cbx99-55.pdf

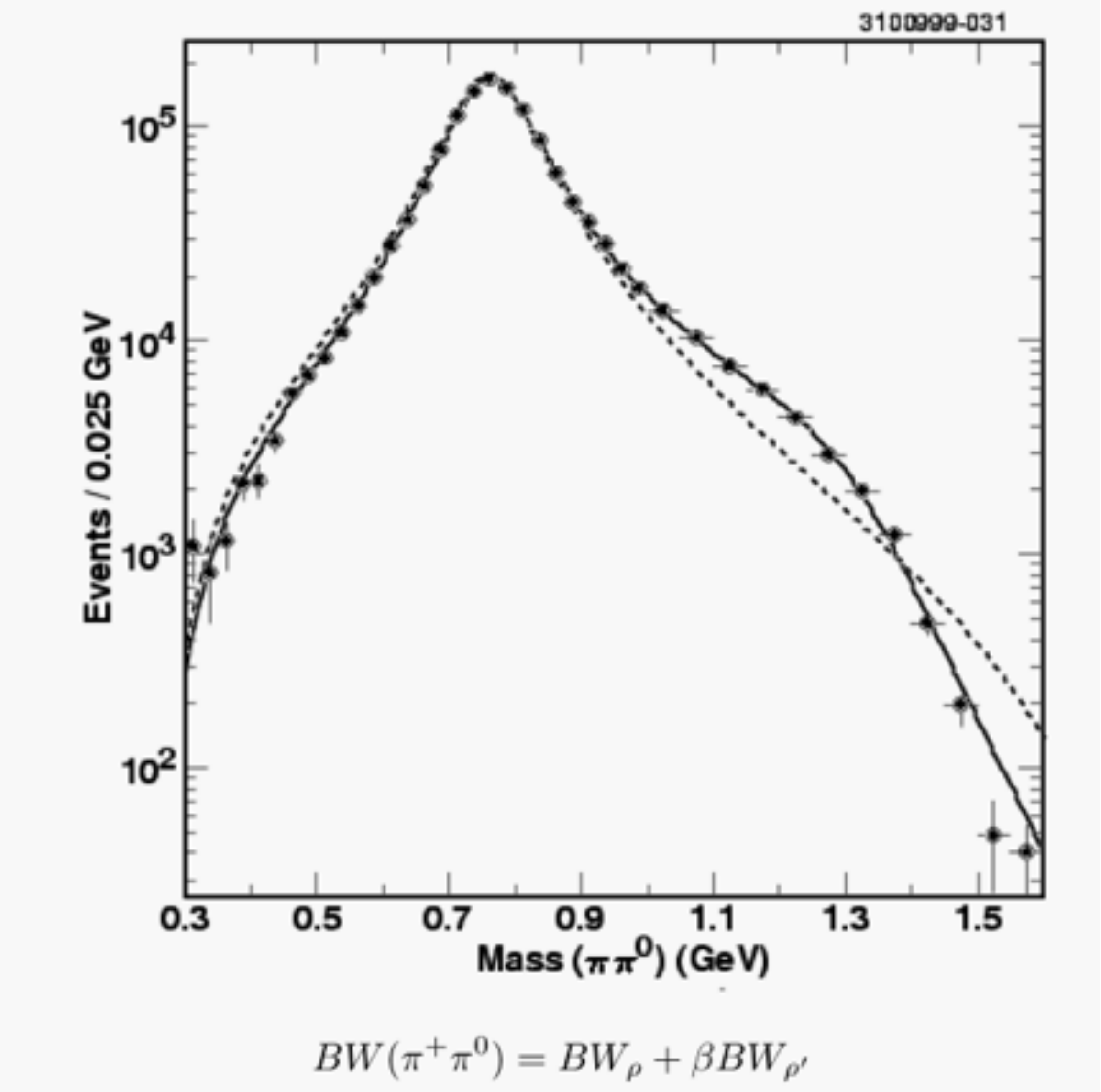
links to a memo on BW's by Alan Weinstein, Caltech.

Interference with higher resonances

Data from tau decays to two pions at CLEO.

Dotted line is without a rho'. Solid line with. The data clearly demands the rho'.

(Feel free to look up rho and rho' in the PDG)



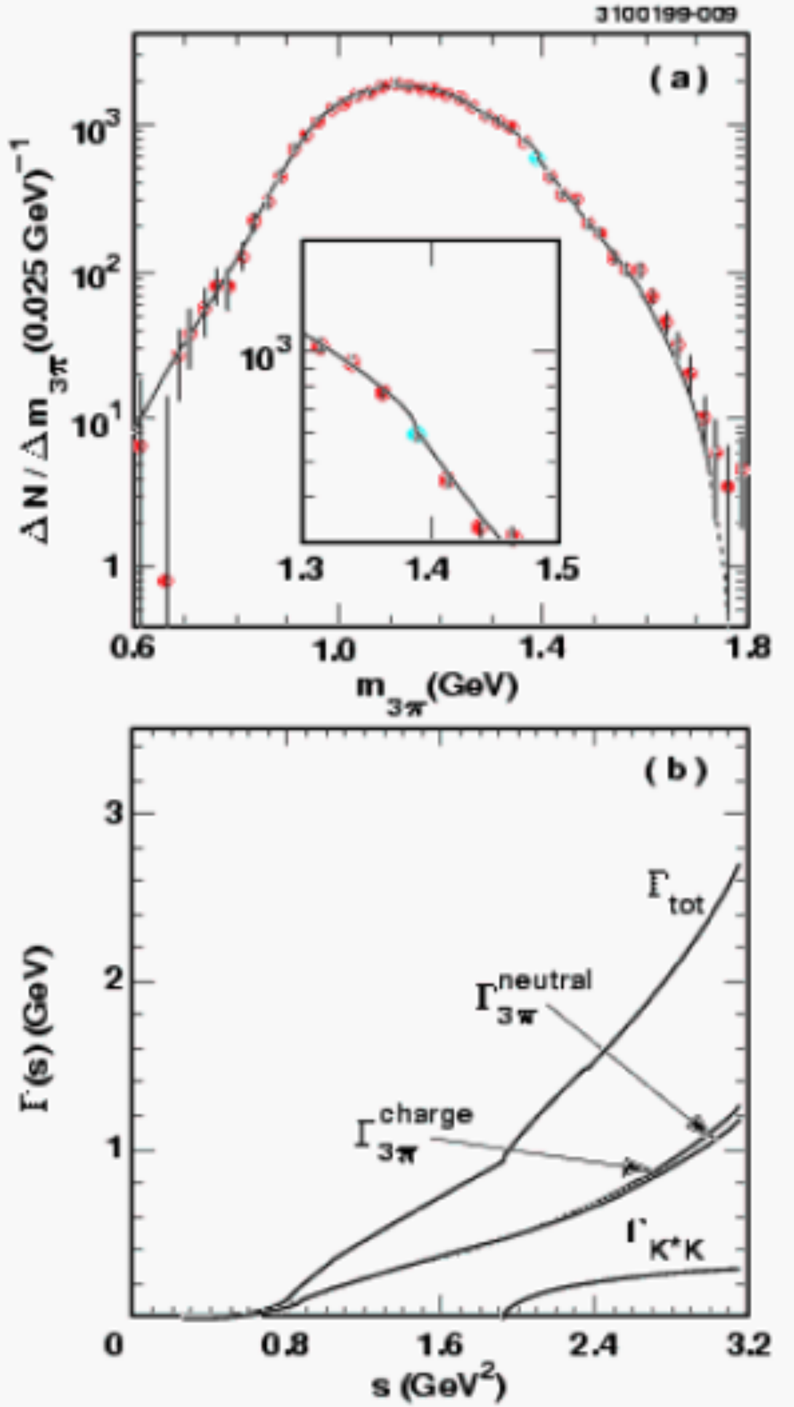
Mass dependent width

Data from tau decays to three pions at CLEO.

The data requires that one allows for a K^*K partial width once allowed kinematically.

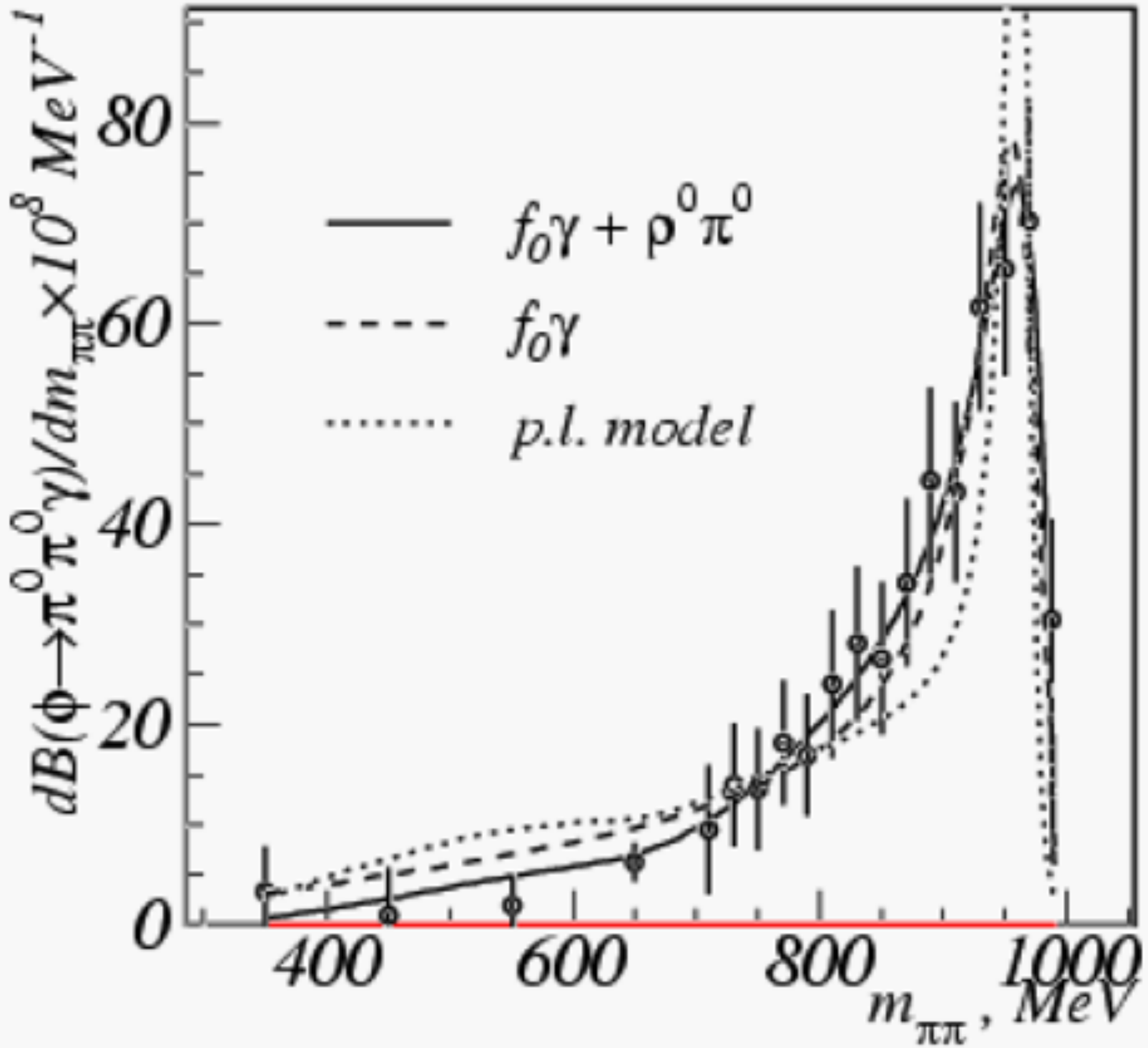
However, even without it, the total width increases significantly as a function of 3-pion invariant mass.

(Feel free to look up the a_1 in PDG)



$\phi(1020) \to f_0(980)\gamma$

Lineshape sculpting due to phase space constraints.



Note: The f_0 is actually wider on the high side than the low side because of the KK kinematic threshold!