Neutrino Mass and Direct Measurements

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0.1 Introduction

The subject of neutrino mass underlies many of the current leading questions in particle physics. Why is there more matter than anti-matter? There are some indications that understanding the mechanism of neutrino mass could help solve this. Why are the neutrinos so light? Figure 1 is a plot of the neutrino mass compared to the mass of the other charged leptons. The neutrino mass is roughly a factor of a million less than the other particles, which only differ in mass by at most a thousand.

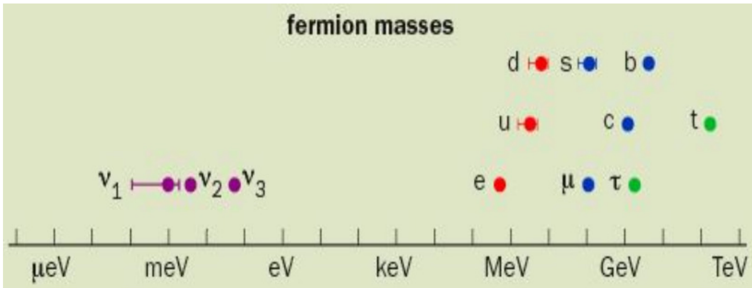

Figure 1: Scale of the neutrino mass compared to the masses of the other charged fermions.

We will discuss briefly how some of these questions may be solved, and discuss general properties of attempts to directly or indirectly measure the mass of the neutrino.

0.2 Lagrangian Formulation of Quantum Field Theory

Standard mechanics in physics can be described in terms of the Calculus of Variation, in which the equation of motion of a body is determined by requiring the *action* to be stationary. The action is defined to be

$$S = \int_{t_0}^{t_1} L(t, x(t), \frac{dx(t)}{dt})dt$$

The quantity in the integral is known as the *Lagrangian*. In general it is a function of the energy of the particle :

$$L = T - V = \frac{1}{2}mv^2 - V$$

or the kinetic energy of the particle minus the potential energy of the particle. It can be shown that minimisation of the action is equivalent to solving the *Euler-Lagrange equation* :

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = 0$$

In Quantum Field Theory (QFT) particles are descibed by fields, with spatial and temporal dependendencies, $\phi_\mu(\mathbf{x}, t)$. We can still define Lagrangians of the fields, however, and the Euler-Lagrange equation still holds. However, since QFT is a relativistic theory, we have to treat space and time the same way, so the time derivative seen in the simple mechanical example must be replaced by a derivative over space and time, and the coordinate derivative is replaced by the derivative of the fields :

$$\frac{\partial L}{\partial(\partial_\mu \phi)} = \frac{\partial L}{\partial \phi}$$

where L is the Lagrangain[1]

The solution to the Euler-Lagrange equation must be the equation of motion of the particle. We know the equation of motion of a spin-1/2 fermion is the Dirac equation so, working backwards, we can guess that the Lagrangian of a free fermion looks like

$$L = \overline{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

Consider the $\overline{\psi}$ field. Using this Lagrangian

$$\frac{\partial L}{\partial(\partial_\mu \overline{\psi})} = 0$$

and

$$\frac{\partial L}{\partial_\mu \overline{\psi}} = (i\gamma^\mu \partial_\mu - m)\psi$$

so, the solution of the Lagrangian gives

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

which is the Dirac equation.

0.2.1 Dirac Mass

Mass is included in the Standard Model through the *Dirac mass term* in the Lagrangian

$$m\overline{\psi}\psi$$

If we take the Dirac spinor ψ and decompose it into it's left- and right-chiral states we we can rewrite this as

$$\begin{aligned} m\overline{\psi}\psi &= m\overline{(\psi_L + \psi_R)}(\psi_L + \psi_R) \\ &= m\overline{\psi_L}\psi_R + m\overline{\psi_R}\psi_L \end{aligned}$$

since we have shown previously that $\overline{\psi_L}\psi_L = \overline{\psi_R}\psi_R = 0$.

The important point to note is that a non-zero Dirac mass requires a particle to have both a left- and right-handed chiral state : in fact the Dirac mass can be viewed as being the coupling constant between the two chiral components. As it stands this formalism is not gauge invariant. To fully integrate this into the Standard Model we need something like the Higgs particle, which can couple with both chiralities. The coupling constant (mass) then becomes more related to the Higgs vacuum expectation value through the symmetry breaking procedure.

Since we know the neutrino is *chirally* left-handed, there is good reason to suppose that the neutrino must be massless, as there does not seem to be a right-handed state for the mass term to couple to. However, we now know that the neutrino *does* have a small mass, so either there must be a right-handed neutrino which *only* shows up in the standard model to give the neutrino mass, but

[1]For more information see Dr Kreps' Gauge Field course.

otherwise cannot be observed as the weak interaction doesn't couple to it, or there is some other sort of mass term out there.

By definition Dirac particles come in four distinct types : left- and right-handed particles, and left- and right-handed antiparticles, and are therefore represented by a 4-component Dirac spinor. As we will see in the next section, charged fermions can only have a Dirac type mass. For neutral fermions, however, this need not be the case. Such particles could also have what is known as a Majorana mass term.

0.2.2 Majorana Mass

Charge Conjugation

A more general theory is based upon a particle's wavefunction, ψ, and it's charge conjugated partner, ψ^c. Recall that the charge conjugation operator, C, turns a particle state into an anti-particle state, by flipping the sign of all the relevant quantum numbers

$$C|\psi >= |\overline{\psi} >$$

One can show that the form for the charge conjugation operator, using Dirac gamma matrices, is

$$C = i\gamma^2\gamma^0$$

with the following properties :

$$\begin{aligned} C^\dagger &= C^{-1} \\ C^T &= -C \\ C{\gamma_\mu}^T C^{-1} &= -\gamma_\mu \\ C{\gamma^5}^T C^{-1} &= \gamma^5 \end{aligned}$$

If ψ is the spinor field of a free neutrino, then the charge-conjugated field ψ^c can be shown to be (in the Dirac representation of the gamma matrices, anyway)

$$\psi^c = C\psi^*$$

where ψ^* is the complex conjugate spinor.

The M ajorana mechanism

Note: The details of the current calculations given here will not be assumed for the exam.

We have seen that the Dirac mass mechanism requires the existance of a sterile right-handed neutrino state. In the early 1930's, a young physicist by the name of Ettore Majorana wondered if he could dispense with this requirement and construct a mass term using only the left-handed chiral state.

Let us first split the Dirac Lagrangian up into its chiral components

$$\begin{aligned} L &= \overline{\psi}(i\gamma^\mu\partial_\mu - m)\psi \\ &= (\overline{\psi_L} + \overline{\psi_R})(i\gamma^\mu\partial_\mu - m)(\psi_L + \psi_R) \\ &= \overline{\psi_L} i\gamma^\mu\partial_\mu\psi_L - m\overline{\psi_L}\psi_L + \overline{\psi_L} i\gamma^\mu\partial_\mu\psi_R - m\overline{\psi_L}\psi_R + \\ &\quad \overline{\psi_R} i\gamma^\mu\partial_\mu\psi_L - m\overline{\psi_R}\psi_L + \overline{\psi_R} i\gamma^\mu\partial_\mu\psi_R - m\overline{\psi_R}\psi R \\ &= \overline{\psi_R} i\gamma^\mu\partial_\mu\psi_R + \overline{\psi_L} i\gamma^\mu\partial_\mu\psi_L - m\overline{\psi_R}\psi_L - m\overline{\psi_L}\psi_R \\ &= \overline{\psi_R}(i\gamma^\mu\partial_\mu - m\overline{\psi_L})\psi_R + \overline{\psi_L}(i\gamma^\mu\partial_\mu - m\overline{\psi_R})\psi_L \end{aligned}$$

Since,

$$\begin{aligned} \overline{\psi_R}\gamma^\mu\partial_\mu\psi_L = \overline{\psi}P_L\gamma^\mu\partial_\mu P_L\psi = \overline{\psi}\gamma^\mu\partial_\mu P_R P_L\psi &= 0 \\ \overline{\psi_L}\gamma^\mu\partial_\mu\psi_R &= 0 \\ m\overline{\psi_R}\psi_R &= 0 \\ m\overline{\psi_L}\psi_L &= 0 \end{aligned}$$

Using the Euler Lagrange equation we can look for independent equations of motion for the left- and right-handed fields, ψ_L and ψ_R. We obtain two coupled Dirac Equations

$$\begin{aligned} i\gamma^\mu\partial_\mu\psi_L &= m\psi_R \\ i\gamma^\mu\partial_\mu\psi_R &= m\psi_L \end{aligned}$$

The mass term couples the two equations. If the field were massless then we have the Weyl equations

$$\begin{aligned} i\gamma^\mu\partial_\mu\psi_L &= 0 \\ i\gamma^\mu\partial_\mu\psi_R &= 0 \end{aligned}$$

The neutrino is now described using only two independent two-component spinors which turn out to be helicity eigenstates and describe two states with definite and opposite helicity. These correspond to the left-handed neutrino and the right-handed neutrino. Since the right-handed neutrino field does not exist, then we just describe the neutrino with a single massless left-handed field and that is that. This is the usual formulation of the Standard Model.

Ettore Majorana wondered if he could describe a *massive* neutrino using just a single left-handed field. At first glance this is impossible as you need the right-handed field to construct a Dirac mass term. Majorana, however, found a way.

Let's take the second equation. We want to try to make it look like the first by finding an expression for ψ_R in terms of ψ_L. To begin with, let's take the hermitian conjugate of the second equation

$$\begin{aligned} (i\gamma^\mu\partial_\mu\psi_R)^\dagger &= m\psi_L^\dagger \\ -i\partial_\mu\psi_R^\dagger\gamma^{\mu\dagger} &= m\psi_L^\dagger \end{aligned}$$

Multiplying on the right by γ^0 we get

$$-i\partial_\mu\psi_R^\dagger\gamma^{\mu\dagger}\gamma^0 = m\psi_L^\dagger\gamma^0$$

One of the properties of the γ matrices is $\gamma^0\gamma^{\mu\dagger}\gamma^0 = \gamma^\mu$. Multiplying on the left by γ^0 and remembering that $(\gamma^0)^2 = 1$, we have $\gamma^{\mu\dagger}\gamma^0 = \gamma^0\gamma^\mu$. Using this

$$-i\partial_\mu\psi_R^\dagger\gamma^0\gamma^\mu = m\psi_L^\dagger\gamma^0$$

and therefore,

$$-i\partial_\mu\overline{\psi_R}\gamma^\mu = m\overline{\psi_L}$$

We want this to have the same structure as the first equation, but that negative sign out the front and the wrong position of the γ^μ matrix is spoiling this. We can deal with this by taking the transpose

$$\begin{aligned} -i[\partial_\mu \overline{\psi_R}\gamma^\mu]^T &= m\overline{\psi_L}^T \\ -i\gamma^{\mu T}\partial_\mu \overline{\psi_R}^T &= m\overline{\psi_L}^T \end{aligned}$$

and using the property of the charge conjugation matrix that $C\gamma^{\mu T} = -\gamma^\mu C$ we get

$$i\gamma^\mu \partial_\mu C\overline{\psi_R}^T = mC\overline{\psi_L}^T$$

This equation has the same structure as the first if we require that the right handed component of ψ is

$$\psi_R = C\overline{\psi_L}^T$$

This assumption requires that $C\overline{\psi_L}^T$ is actually right-handed. Is this true? Well, if the field is right-handed then applying the left-handed chiral projection operator, $P_L = \frac{1}{2}(1-\gamma^5) : P_L\psi_R = 0$. Using the properties of the charge conjugation matrix, $P_L C = CP_L^T$, we have

$$P_L(C\overline{\psi_L}^T) = CP_L^T\overline{\psi_L}^T = C(\overline{\psi_L}P_L)^T$$

Now,

$$\begin{aligned} \overline{\psi_L}P_L &= (P_L\psi)^\dagger\gamma_0 P_L \\ &= \psi^\dagger P_L^\dagger \gamma_0 P_L \\ &= \psi^\dagger P_L \gamma_0 P_L \\ &= \psi^\dagger \gamma^0 P_R P_L \\ &= 0 \end{aligned}$$

since $P_L^\dagger = P_L$, $\{\gamma^5, \gamma_0\} = 0$ and $P_R P_L = \frac{1}{4}(1+\gamma^5)(1-\gamma^5) = \frac{1}{4}(1-\gamma^{5^2}) = 0$.

So, yes, if we define the right-handed field $\psi_R = C\overline{\psi_L}^T$, then we can write the Dirac equation *only* in terms of the left-handed field ψ_L. The Majorana field, then becomes

$$\psi = \psi_L + \psi_R = \psi_L + C\overline{\psi_L}^T = \psi_L + \psi_L^C$$

where we've defined the *charge-conjugate field*, $\psi_L^C = C\overline{\psi_L}^T$.

What does this imply? Well, let's take the charge conjugate of the Majorana field :

$$\psi^C = (\psi_L + \psi_L^C)^C = \psi_L^C + \psi_L = \psi$$

That is, the charge conjugate of the field is the same as the field itself, or more prosaically, a Majorana particle is it's own anti-particle.

What sort of particle can be Majorana - well, clearly it must be neutral, as the charge conjugation operator flips the sign of the electric charge. Any charged fermion therefore will not be identical to its antiparticle. In fact, the only neutral fermion that could be a Majorana particle is the neutrino.

Majorana mass term

We saw above that the mass term in the Lagrangian couples left- and right-handed neutrino chiral states : $L^D = -m\overline{\nu_R}\nu_L$. If the particle is Majorana, We can also form a mass term just with the left-handed component. In this case, the right-handed component is $\nu_L^C = C\overline{\nu_L}^T$

$$L_L^M = -\frac{1}{2}m\overline{\nu_L^C}\nu_L$$

The factor of a half there is to account for double-counting since the hermitian conjugate is identical.

Lepton Number violation

The Majorana term couples the antineutrino to the neutrino component. Dirac neutrinos have lepton number $L = +1$ and antineutrinos have lepton number $L = -1$. Since Majorana neutrinos are the same as their antiparticle it is impossible to give such an object a conserved lepton number. Indeed, interactions involving Majorana neutrinos generally violate lepton number conservation by $\Delta L = \pm 2$.

Talking about Majorana neutrinos in terms of neutrino and anti-neutrino can get confusing and misleading. If the neutrino is Majorana, then there is only one particle : the neutrino. When this neutrino couples to the weak current it can either produce a negatively charged lepton, if the left-handed component of the Majorana field interacts with a W^+, or a positively charged lepton if the right-handed component interacts with a W^-.

0.2.3 The Seesaw mechanism

We've seen that if only the left-handed chiral field, ν_L, exists then there can be no Dirac-type mass term. In this case the neutrino Lagrangian can contain the Majorana mass term

$$L_{Maj}^L = -\frac{1}{2}m_L\overline{\nu_L^C}\nu_L + h.c.$$

and the neutrino is a Majorana particle. Unfortunately, given the build-up I've been setting up, such a term cannot actually exist in the standard model, and so $m_L = 0$[2]

Hence, whatever one does, that the neutrino has a mass must imply either (i) there is something really odd we haven't thought of (never discount this possibility) or (ii) a right-handed chiral neutrino field exists that only interacts with gravity and the Higgs mechanism.

We'll set m_L equal to zero below, but let's first assume that we can write a left-handed Majorana mass term $L_L^M = \frac{1}{2}m_L\overline{\nu_L^C}\nu_L + h.c.$. If we're resigned to have a right-handed neutrino field as well, which we'll call ν_R, we can write a Dirac mass term $L_D = \overline{\nu_R}\nu_L + h.c.$ and a further *right*-handed Majorana field $L_R^M = \frac{1}{2}m_R\overline{\nu_R^C}\nu_R + h.c.$. We also have the charge-conjugate fields, ν_L^C and ν_R^C. These can form another Dirac mass term, $m_D\overline{\nu_L^C}\nu_R^C$. The mass for this term must be the same as for the other Dirac mass term, as the total Majorana fields are $\nu_L + \nu_L^C$, and $\nu_R^C + \nu_R$. Note that the *h.c.* just means the hermitian conjugate.

[2] The reason is complicated and bound up with the Higgs mechanism. In general though, when one includes the Higgs, one obtains interaction terms in the standard model Lagrangian that look like $\overline{\nu_L^C}H\nu_L$ where H is the Higgs field. The left-handed neutrino field in the standard model has weak isospin $I = \frac{1}{2}$ and hypercharge $Y = -1$. This interaction term must cancel the quantum numbers to be gauge invariant, and so the H field would need to have weak isospin $I = -1$ and hypercharge $Y = -2$. Such a field does not exist in the standard model so one can't fit a left-handed neutrino term like this into the model. On the other hand, a right-handed neutrino field can exist.

If one includes all these terms, then the most general mass term one can write down is

$$\begin{aligned}2L_{mass} &= L_L^D + L_R^D + L_L^M + L_R^M + h.c. \\ &= m_D\overline{\nu_R}\nu_L + m_D\overline{\nu_L^C}\nu_R^C + m_L\overline{\nu_L^C}\nu_L + m_R\overline{\nu_R^C}\nu_R + h.c.\end{aligned}$$

[3] which can be written as a matrix equation

$$L_{mass} \sim \begin{pmatrix}\overline{\nu_L^C} & \overline{(\nu_R)}\end{pmatrix}\begin{pmatrix}m_L & m_D \\ m_D & m_R\end{pmatrix}\begin{pmatrix}\nu_L \\ \nu_R^C\end{pmatrix} + h.c.$$

with the mass matrix

$$M = \begin{pmatrix}m_L & m_D \\ m_D & m_R\end{pmatrix}$$

Now, let's consider this a moment. We have expressed the mass Lagrangian in terms of the *chiral* fields : ν_L and ν_R. These fields clearly do not have a definite mass because of the existence of the off-diagonal term, m_D, in the mass matrix. That means that these fields are not the mass eigenstates, and do not correspond to the *physical* particle - which must have a definite mass. This should be no concern - it just means that the flavour eigenstate which couples to the W and Z bosons is a superposition of the massive neutrino states. One could say, for example and pulling some numbers out of the air, that the massive neutrino, ν_m, has a 95% chance of interacting as a ν_L and a 5% chance of interacting as a ν_R. Then one could write the state as $|\nu_m >= \sqrt{0.95}|\nu_L > +\sqrt{0.05}|\nu_R >$. Hence the mass state, ν_m is a mix of the flavour eigenstates. The same thing happens in the quark sector when one talks about the CKM matrix. In the quark case, we noted that the down and strange fields with definite mass are the d, s and b states. The states which couple to the weak interaction are the d', s' and b' states, and these are related by a unitary matrix

$$\begin{pmatrix}d' \\ s' \\ b'\end{pmatrix} = \begin{pmatrix}V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb}\end{pmatrix}\begin{pmatrix}d \\ s \\ b\end{pmatrix}$$

The primed states couple to the W and Z, whereas the unprimed states have definite mass. The same thing happens in this case

Let's call these mass eigenstates ν_1 and ν_2. In order to find the mass of these states, we need to rewrite the Lagrangian in terms of ν_1 and ν_2, which will give us a diagonal mass matrix

$$M' = \begin{pmatrix}m_1 & 0 \\ 0 & m_2\end{pmatrix}$$

The procedure for doing this is standard. One looks for a unitary matrix, U, which transforms the left-handed chiral fields into left-handed components of fields with definite mass

$$\begin{pmatrix}\nu_L \\ \nu_R^C\end{pmatrix} = U\begin{pmatrix}\nu_{1,L} \\ \nu_{2,L}\end{pmatrix}$$

This then implies that

$$U^\dagger M U = M'$$

[3]Note here that the Dirac mass term coming from the Dirac equation is $m_D(\overline{\nu_R}\nu_L + \overline{\nu_L}\nu_R)$. The two terms are identical so we can just used $\frac{1}{2}m_D\overline{\nu_R}\nu_L$. Similarly for the conjugate fields.

Such a matrix always exists (that's a standard theorem in linear algebra) - the masses of the massive neutrinos are then m_1 and m_2 respectively. This transformation then implies that the left-handed mass states $\nu_{1,L}$ and $(\nu_{2,R})^c$ are expressed in terms of the chiral states ν_L and $(\nu_R)^c$ as

The only 2x2 unitary matrix is the standard rotation matrix

$$\begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$$

so

$$\begin{aligned} \nu_L &= cos\theta\nu_{1,L} - sin\theta\nu_{2,L} \\ (\nu_R)^c &= sin\theta\nu_{1,L} + cos\theta\nu_{2,L} \end{aligned}$$

Diagonalisation of a square matrix

As a reminder, the procedure to diagonalise a square matrix is as follows If our matrix is of the form

$$\begin{pmatrix} A & B \\ B & C \end{pmatrix}$$

1. Find the eigenvalues of the matrix by solving the characteristic polynomial

$$det\left[\begin{pmatrix} A-\lambda & B \\ B & C-\lambda \end{pmatrix}\right] = (A-\lambda)(C-\lambda) - B^2 = 0$$

2. For each eigenvalue, λ_1, λ_2, find the relevant eigenvector $\mathbf{v}_1$, $\mathbf{v}_2$.
3. The matrix we want is

$$U = \begin{pmatrix} \mathbf{v_1} & \mathbf{v_2} \end{pmatrix}$$

For example, consider the matrix

$$A = \begin{pmatrix} 7 & 2 \\ 2 & 1 \end{pmatrix}$$

The characteristic equation is $(7-\lambda)(1-\lambda) - 4 = 0$, with eigenvalues $\lambda_{1,2} = 4 \pm 2\sqrt{13}$. For the first eigenvalue, $\lambda_1 = 7.61$, we find an eigenvector of

$$\mathbf{v_1} = \begin{pmatrix} 0.957 \\ 0.29 \end{pmatrix}$$

or the second eigenvalue, $\lambda_2 = 0.394$ we have an eigenvector of

$$\mathbf{v_2} = \begin{pmatrix} -0.29 \\ 0.957 \end{pmatrix}$$

The transformation matrix we want is

$$U = \begin{pmatrix} 0.957 & -0.29 \\ 0.29 & 0.957 \end{pmatrix}$$

Note, now that if we form that matrix product

$$U^{-1}AU = \begin{pmatrix} 0.957 & 0.29 \\ -0.29 & 0.957 \end{pmatrix} \begin{pmatrix} 7 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 0.957 & -0.29 \\ 0.29 & 0.957 \end{pmatrix} = \begin{pmatrix} 7.61 & 0 \\ 0 & 0.394 \end{pmatrix}$$

If we apply this procedure to the mass matrix, M, we find that the masses $m_{1,2}$ can be expressed in terms of m_L,m_R and m_D using the characteristic equation as

$$m_{1,2} = \frac{1}{2}\left[(m_L + m_R) \pm \sqrt{(m_L - m_R)^2 + 4m_D^2}\right]$$

We can choose different values for m_L, m_R and m_D and this will give us different physical masses, but the interesting behaviour occurs if one chooses $m_L = 0$ and $m_R >> m_D$. As we have seen, the standard model explicitly forbids the left-handed Majorana term ($m_L = 0$) but says nothing about the right-handed Majorana term, so this choice of parameters is sensible. If we makes this choice, we get the mass of the ν_1 field to be

$$m_1 = \frac{m_D^2}{m_R}$$

and the mass of the ν_2 field to be

$$m_2 = m_R\left(1 + \frac{m_D^2}{m_R^2}\right) \approx m_R$$

Notice now that if we have a neutrino with a mass, m_2, that is very large, then the mass of the other neutrino, m_1, is very small, because of the suppression provided by the $\frac{1}{m_R}$ factor.

If we try to find expressions for our mass eigenstates, we find that if m_R is very large, $\nu_1 \sim (\nu_L + \nu_L^c) - \frac{m_D}{m_R^2}(\nu_R + \nu_R^c)$ and $\nu_2 \sim (\nu_R + \nu_R^c) + \frac{m_D}{m_R^2}(\nu_L + \nu_L^c)$. That is, ν_1 is mostly our familiar left-handed light Majorana neutrino and ν_2 is mostly the heavy sterile right-handed partner.

This is the famous *see-saw mechanism*. It provides an explanation for the question of why the neutrino has a mass so much smaller than the other charged leptons. The charged leptons are Dirac particles and therefore have a Dirac mass on the order of 1 MeV (or so). Suppose the Dirac mass of the neutrino is around the same value ($m_D \approx 1\ MeV$) like all the other particles. Then if the mass of the heavy partner is around 10^{15} eV, the mass of the light neutrino will be in the meV range, as we now know it is.

This is the only natural explanation of the relative smallness of the neutrino mass we currently have. It requires that the neutrino be a Majorana particle, and that there exists an extremely heavy partner to the neutrino with a mass too large for us to be able to create it. This may seem a bit of a stretch, but we know that such particles would have been created very early in the universe. They no longer exist as stable particles, as they have decayed to lighter states as the universe cooled, but due to the uncertainty principle, could exist for the short time necessary to generate mass.

The possibility of the existence of these heavy neutrinos also have given rise to another intriguing idea called *leptogenesis* which is intrinsically linked to the question of the matter-antimatter asymmetry. The idea is that these very heavy neutrinos, which are Majorana particles, decayed as the universe cooled into lighter left-handed neutrinos or right-handed antineutrinos, along with Higgs bosons, which themselves decayed to quarks. If the probability of one of these heavy neutrinos to decay to a left-handed neutrino was slightly different than the probability to decay to a right-handed anti-neutrino, then there would be a greater probability to create quarks than anti-quarks and the universe would be matter dominated. More formally, it is thought that the quantum number $B - L$, where B is the baryon number of the universe and L is the lepton number, must be conserved. If there was some violation of L in the decays of the heavy Majorana neutrinos, this would manifest as a violation in B, and hence the missing anti-matter problem could actually arise from CP violation in the neutrinos. Although there is no direct connection between CP violation in the heavy neutrinos and CP violation in the light neutrinos, this idea still motivates the current attempt to measure CP violation in the light neutrinos at today's long baseline neutrino experiments.

This argument is one of the reasons why it is important to determine whether the neutrino is a Majorana particle or not.

0.2.4 Implications of Neutrino Mass

Either

- the neutrino is a Dirac particle, and hence there must be a right-handed chiral neutrino state, which does not interact with matter. This is a so-called *sterile* neutrino.
- the neutrino is a Majorana particle. In this case, the neutrino and the antineutrino are identical. The mass term directly couples the left-handed neutrino with the right-handed antineutrino, which implies that the neutrino must have mass. Furthur, such an interaction implies that lepton number is violated by 2.
- If the neutrino is Majorana, then it is possible to explain the very low value of the neutrino mass compared to the quark and charged lepton masses at the expense of the introduction of another very heavy Majorana neutrino though the see-saw mechanism. CP violation in the decays of this heavy neutrino in the early universe could have created the baryon asymmetry we see today.

0.2.5 What you need to know

- The difference between Dirac and Majorana particles and why only the neutrino could be a Majorana fermion
- the implications of the neutrino being a Dirac or Majorana particle.
- what the see-saw mechanism is and how it explains the low value of the light neutrino mass.

0.3 Direct Neutrino Mass Measurements

Attempts at measuring the mass of the neutrinos generally involve studying well-known decays of particles which decay to a neutrino and some charged particles. If the momentum of these charged particles can be measured, and the mass of the decaying parent, and the decay products is well known, then energy-momentum conservation can, in principle, allow one to determine the mass of the outgoing neutrino.

Since the neutrino mass is so small, experiments of this type are usually extremely difficult. Any systematic error on the order of the neutrino mass could be interpreted as a non-zero neutrino mass, so the experiments must be very well understood.

0.3.1 Direction measurement of the electron neutrino mass

The standard method of measuring the mass of the electron neutrino is to use β-decay. The underlying mechanism for β-decay is

$$n \to p + e^- + \overline{\nu_e}$$

The general theory of β-decay describes the shape of the energy of the electron emitted from the decay by the equation

$$\frac{dN}{dE_e} = Cp(E + m_e)(E_0 - E_e)\sqrt{(E_0 - E_e)^2 - m_\nu^2}F(E_e)\theta(E_0 - E_e - m_\nu)$$

where C is just a normalisation constant, m_e is the electron mass, E_e is the electron energy, and E_0 is the maximum allowable energy for the electron from the decay kinematics, and is called the *end point*. The function $F(E_e)$ is called the Fermi function, and takes it account the interactions of the electron with the electromagnetic field of the daughter nucleus. The final term, $\theta(E_0 - E_e - m_\nu)$, just imposes energy conservation. This spectrum can be seen in 2(a). A non-zero neutrino mass has the effect of changing the slope of the curve slightly, and also changing the maximum allowable energy given to the electron. This is shown in 2(b) which shows the difference in the shape of the spectrum between a zero neutrino mass and a mass of 1 eV.

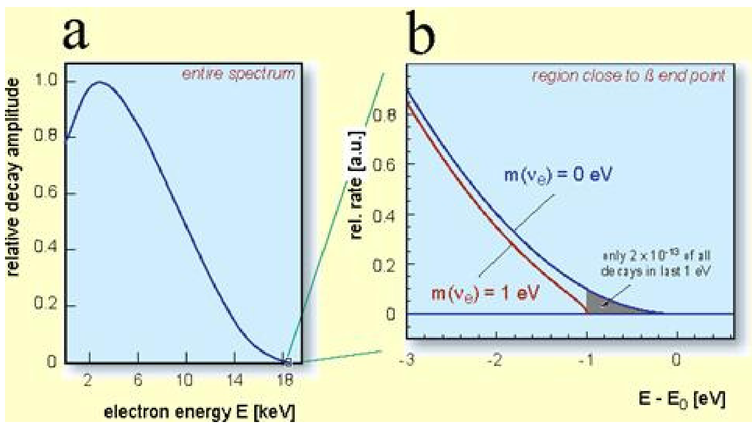

Figure 2: (a) The shape of the electron energy spectrum in β-decay. (b) The effect of a non-zero neutrino mass on the endpoint of the spectrum.

Direct measurements search for the slight difference in shape right at the endpoint of the spectrum which may be indicative of a non-zero neutrino mass. They are difficult experiments to do, and can only be done under a restrictive set of conditions :

- The number of electrons near the end point of the spectrum is usually small, so the statistical error is large. In general, an isotope which has a small end point is better as the fraction of electrons near the endpoint is larger.
- The detector should have extremely good electron energy resolution.
- The β source shouldn't be *thick*. Once emitted from the decay, the electron could lose energy as it travelled through the source, biasing the measurement. Ideally the source should be a single layer of atoms. However this would lead to a very low event rate. The best thing to do is find a *gaseous* source - this combines low density so minimal energy loss, with a reasonable number of source atoms to decay.
- The end point of the spectrum is sensitive to lots of atomic and nuclear effects, excited state transitions and other rarer effects. When looking for an already rare signal, one wants an isotope which doesn't have as many of these nuclear effects which would distort the endpoint and be difficult to understand.

One of the better source materials for this purpose is gaseous tritium which decays via the channel

$$^3H \rightarrow {}^3He + e^- + \overline{\nu_e}$$

A schematic for the sort of experiment that is used for this measurement is shown in Figure 3

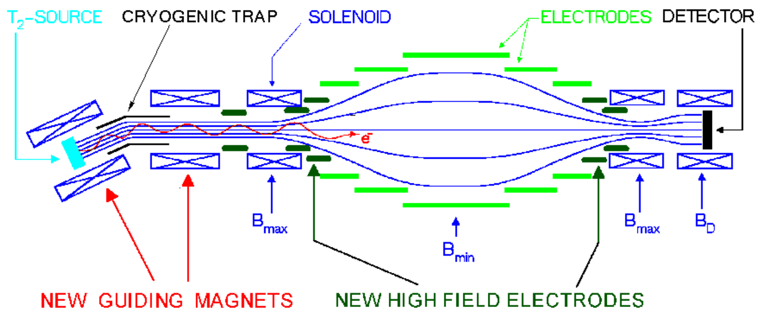

Figure 3: Schematic of a tritium-beta decay experiment.

The tritium is stored at the extreme left. When it β-decays, the emitted electron is guided along the input channel to the large retarding spectrometer just after the solenoid. The spectrometer imposes a retarding electron potential along its length, stopping all but the most energetic electrons. Those electrons with high enough energy to surmount the potential barrier are then guided into the detector at the right. By changing the value of the potential, an integral spectrum of the electrons can be built up and the end point studied for signs of neutrino mass.

The results of these experiments have been to put an upper limit on the mass of the electron neutrino : $m_{\nu_e} < 2.2$ eV at 95% confidence. A new experiment, called KATRIN, is about to start running (in 2012) which aims to decrease this limit to 0.2 eV in 5 years of running. If the electron neutrino mass is above 0.35 eV, KATRIN will measure it to a precision of 5 standard deviations.

0.3.2 Direct measurement of the muon and tau neutrino mass

The only way to directly measure the mass of the muon and tau neutrinos are through the decays of pions and tau. An upper limit on the muon neutrino mass has been made by investigating the decay

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

for decays of pions at rest. This has yielded the upper limit of $m_{\nu_\mu} < 190$keV at 90% confidence, with the dominant error being our imprecise knowledge of the pion mass.

The results for the tau neutrino mass are even worse. The few measurements to be done on this were performed at e^-e^+ colliders using the decay scheme :

$$e^+ + e^- \rightarrow \tau^+ + \tau^-$$

The τ then decayed into the 5 hadron final state

$$\tau \rightarrow \nu_\tau + 5\pi^\pm(\pi^0)$$

The energy of each tau is just half the centre of mass energy of the beam, so is well known. The problem comes in being sure that the detector has recorded, and reconstructed, all of the five pions well and with sufficient precision as to be sensitive to the missing energy and momentum carried away by the ν_τ. The upper limit for the mass of the ν_τ is currently $m_{\nu_\tau} < 18.2$MeV at 95% confidence.

0.3.3 What you should know

- General principles of ν_e mass detection using β decay. What does neutrino mass do to the end point of the electron spectrum? What properties are needed for experiments which try to do this kind of measurement.
- How measurements of the mass of the ν_μ and ν_τ are attempted.

0.4 Neutrinoless Double Beta Decay

As can be seen from the previous section, the extremely small size of the neutrino mass makes direct measurement very difficult.

One alternate method which is under enthusiastic investigation around the work is the study of neutrinoless double beta decay. We will start talking about this by briefly discussing the standard model process of 2 neutrino double beta decay.

Certain rare isotopes are forbidden from decaying through standard beta decay. The mass of such isotopes are usually less than than the daughter isotope after beta decay

$$m(Z, A) < m(Z + 1, A)$$

These isotopes are energetically allowed, however, to decay via *double beta decay*, a nuclear process whereby the nuclear charge changes by 2 units while the atomic mass is left unchanged

$$(Z, A) \rightarrow (Z + 2, A) + 2e^- + 2\overline{\nu_e}$$

There are only a few isotopes that decay this way (see Table 1). The process is a fourth-order process with a lifetime that is proportional to $(G_F cos\theta_C)^{-4}$. Hence, they are distinguished by extremely long half-lives and rare natural abundances.

$2\nu\beta\beta$ mode	Half life ($\times 10^{24}$ years)
${}^{48}_{20}Ca \rightarrow {}^{48}_{22}Ti$	4.1
${}^{76}_{32}Ge \rightarrow {}^{76}_{34}Se$	40.9
${}^{82}_{34}Se \rightarrow {}^{82}_{36}Kr$	9.3
${}^{96}_{40}Zr \rightarrow {}^{96}_{42}Mo$	4.4
${}^{100}_{42}Mo \rightarrow {}^{100}_{44}Ru$	5.7
${}^{110}_{46}Pd \rightarrow {}^{110}_{48}Cd$	18.6
${}^{116}_{48}Cd \rightarrow {}^{116}_{50}Sn$	5.3
${}^{124}_{50}Sn \rightarrow {}^{124}_{52}Te$	9.5
${}^{130}_{52}Te \rightarrow {}^{130}_{54}Xe$	5.9
${}^{136}_{54}Xe \rightarrow {}^{136}_{56}Ba$	5.5
${}^{150}_{60}Nd \rightarrow {}^{150}_{62}Sm$	1.2

Table 1: Some isotopes which undergo 2 neutrino double-beta decay and the half-life of the transition

This process would have remained a rare but unremarkable process if it were not for the discovery of neutrino mass. It was Ettore Majorana who suggested that, provided the neutrino had mass, and was a Majorana particle, then the process of *zero*-neutrino (or neutrinoless) double beta decay should also be observable

$$(Z, A) \rightarrow (Z + 2, A) + 2e^-$$

Clearly this process violated lepton number conservation by two units. The process continues in two stages. First a neutron decays, emitting a right-handed $\overline{\nu_e}$. This has to be absorbed by a second neutron as a left–handed ν_e (see Figure 4). To fulfil these conditions, the neutrino and anti-neutrino must be the same (hence the neutrino must be a Majorana particle). Further, the chiral right-handed $\overline{\nu_e}$ emitted by the first decay must be able to evolve a left-handed chiral component as it propagates towards the second interaction. The only way it can do this is if the neutrino has a mass. The observation of neutrino-less double beta decay shows both that the neutrino is Majorana and that the process can be used to obtain an estimate of the absolute neutrino mass (unlike neutrino oscillations which cannot, in principle, measure this). The half-life of neutrinoless double beta decay may be expressed as

$$T_{0\nu}^{-1} = G^{0\nu}|M^{0\nu}|^2|m_{ee}|^2$$

where $G^{0\nu}$ is a known phase-space factor which determines how many electrons have the right energies and momenta to participate in this process, $M^{0\nu}$ is a *nuclear matrix element* which describes the actual decays in the nuclear environment, and m_{ee} is a linear combination of the neutrino mass states

$$|m_{ee}| = |\sum_i U_{ei}^2 m_i|$$

with U_{ei} being the relevant elements of the PMNS mixing matrix, and m_i the mass of the mass eigenstates.

Hence, any measurement of the half-life of this process gives one a handle on the scale of the neutrino mass which, when combined with the results of the oscillation experiments, can provide a measurement of the lightest mass eigenstate.

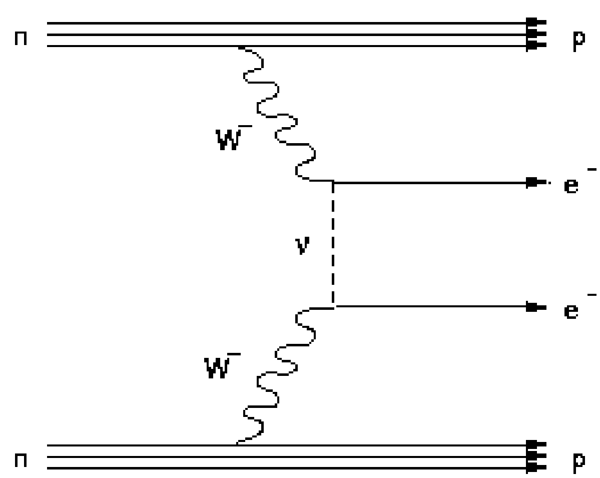

Figure 4: Feynman diagram for neutrino-less double beta decay

An interesting consequence of the mixing of the mass states in the measurement of m_{ee} is that neutrinoless double beta decay can also provide information on whether the mass states are in the normal, or inverted heirarchy. Without going into details, one can show that the values of m_{ee} depend on whether there are two heavy and one light, or one light and two heavy states. Figure 5 shows a plot of m_{ee} as a function of the lightest neutrino mass for the two different mass hierarchy hypotheses:

- The quasi-degenerate region where $m_1 \approx m_2 \approx m_3$. In this case $m_{ee} > \sqrt{\Delta m_{23}^2}$, or $m_{ee} > 0.05eV$.
- The inverted hierarchy scheme where $m_3 << m_1 < m_2$. In this case the effective mass, m_{ee} is bounded above and below : 0.01 eV $< m_{ee} <$ 0.05 eV.
- The normal hierarchy scheme where $m_1 < m_2 << m_3$. In this case $m_{ee} <$ 0.005 eV.

If one can measure m_{ee} to be less than 0.01 eV then one has shown that the heirarchy is normal. This is another reason why there is quite a lot of experimental effort going towards making this measurement.

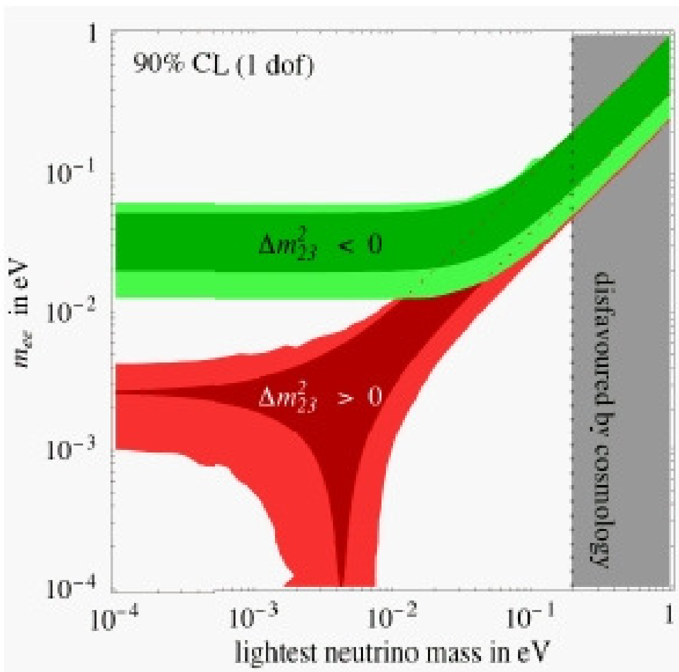

Figure 5: A plot of m_{ee} as a function of the lightest neutrino mass for the two different mass heirarchy hypotheses.

0.4.1 Experimental Conditions for the measurement of $0\nu\beta\beta$ decay

The signal for neutrinoless double beta decay is a spike in the spectrum of the sum of the emitted electrons at a specific energy (the Q-value of the transition). Conversely, for 2ν double beta decay, this spectrum will be continuous from 0 up to the Q-value (see Figure 6).

The sensitivity to m_{ee} scales with the square root of the half-life. The half-life scales as

$$T^{0\nu}_{1/2} \propto a\sqrt{\frac{Mt}{B\Delta E}}$$

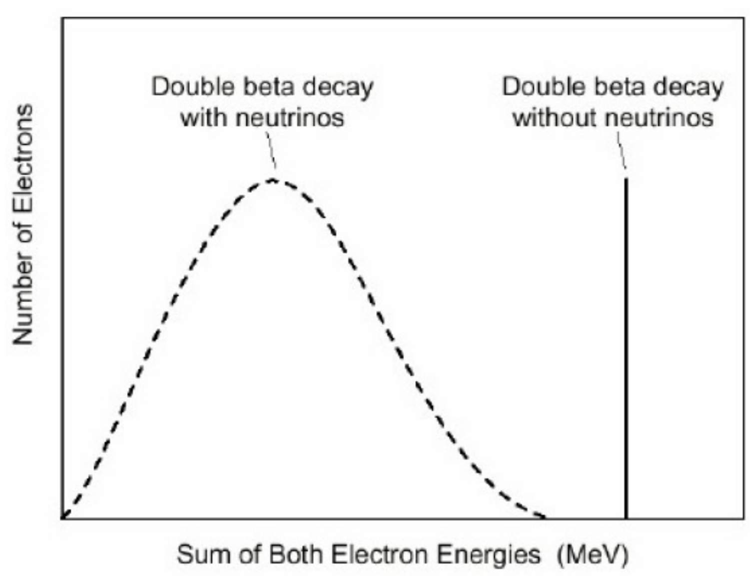

Figure 6: The energy spectrum of the sum of the energies of the emitted electrons for 2ν double beta decay (broad continuous distribution) and 0ν double beta decay (spike at endpoint of the spectrum).

where a is the abundance of the isotope in the source, M is the source mass, t is the measurement time, B is the number of background counts and ΔE is the energy resolution of the detector. This makes sense - clearly a better measurement can be made if there is more isotope to decay, and/or more source material. Alternatively one could just count for longer, put more effort into minimising the background, or make a detector with extremely good energy resolution. There are two types of neutrinoless double beta decay detectors

- the counting experiment : This type of experiment usually utilises the source as the detector. It typically has excellent energy resolution, and can have large source masses, but often only measures the sum energy of the two electrons and therefore tends to have more irreducible background. The two electrons themselves are never observed directly.

- the tracking experiment : In this type of experiment the two electrons are detected independently using tracking and calorimetry. This minimises background, as in 0ν double beta decay the electrons are emitted back-to-back, but the source mass is usually constrained by the detector design.

0.4.2 Counting Experiments

Semiconductor experiments

In this type of experiment, the source material is usually some form of semiconductor. The isotope under investigation is part of the source. When a double beta decay event occurs, the emitted electrons ionise the semiconductor, leading to a cascade of electron/hole pairs that drift to electrodes on the faces of the detector, generating a voltage pulse that can be measured. The advantage of such detectors is that the number of electron/hold pairs is proportional to the energy of the emitted electrons, hence the energy resolution is usually extremely good (2-3% for 1 MeV electrons). However, the detector only measures the sum energy of the two electrons, and hence there is usually a lot of background from other radioactive processes occurring in the source material.

The Heidelberg-Moscow experiment is probably the most famous and controversion semi-conductor based neutrinoless double beta decay experiment. It used 11 kg of Ge enriched to about 80% in ^{76}Ge. It is controversial in that it is the only neutrinoless double beta decay experiment to claim a positive result. Figure 7 shows the sum energy spectrum that they measure. A peak is expected at 2039 keV, and indeed the collaboration claim that one is observed at a significance of 4σ. Unfortunately, there are still a lot of background peaks from other processes occurring in the diode material. Most are known, but the peak at 2030 keV (if it is a peak) is unknown. If true, Heidelberg-Moscow suggests an effective neutrino mass of $m_{ee} = 0.40 \pm 0.17$ eV. However, no confirmation has been found in other experiments, so this is still considered controversial. If one assumes that no signal has been seen, then an upper limit can be put on the effective mass of

$$m_{ee} < 0.35 \text{ eV}$$

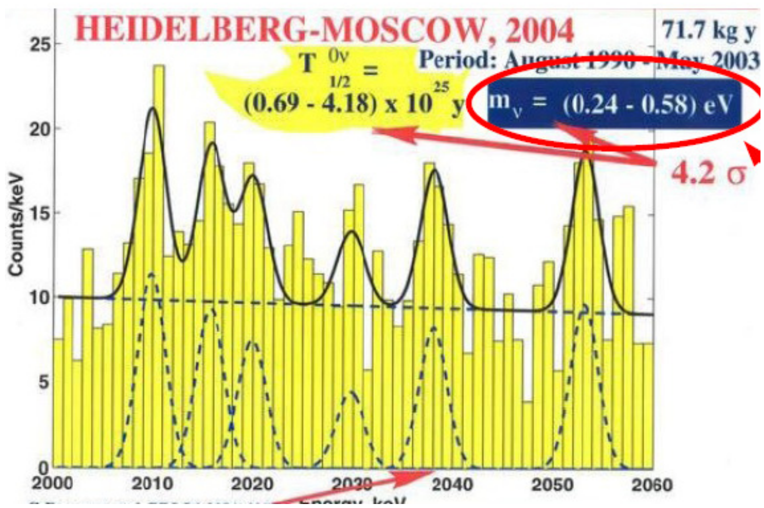

Figure 7: The controversial discovery claim of the Heidelberg-Moscow experiment. The claim is that the peak at 2040 keV represents a 4σ discovery of neutrinoless double beta decay.

Cryogenic experiments

This type of experiment works much the same as the semiconductor experiments. The emission of the electrons from the double beta decay raises the temperature of the material by a tiny amount. If the detector is maintained at a very low temperature (on the order of mK) then this temperature increase can be measured and used to determine the summed electron energy. The leading experiment of this type is Cuoricino which is made of 40 kg of TeO_2 held at 8 mK. A neutrinoless double beta decay event would raise the temperature of the crystal by about 10 μ K. This has been running since 2003 and has not seen evidence of neutrinoless double beta decay. It has set an upper bound of $m_{ee} < 0.68eV$.

Future

There are a large number of experiments planned to start running in the next 10 years. They all aim for a source mass of more than a hundred kilograms (hard when the actual isotope you want is quite rare). A non-exhaustive selection are listed in Table 2

Experiment	Isotope	Source mass	Mass sensitivity	Start date
GERDA	^{76}Ge	40 kg	$< 0.02eV$	2010
Majorana	^{76}Ge	500 kg	$< 0.01eV$	2011
Cuore	^{130}Te	200 kg	$< 0.02eV$	2011
Cuore	^{130}Te	200 kg	$< 0.02eV$	2011
Candles	^{48}Ca	0.5	$< 0.5eV$	2009

Table 2: A non-exhaustive list of neutrinoless double beta decay experiments under construction and their mass sensitivities.

0.4.3 Tracking experiments

Counting experiments try to maximise their sensitivity by maximising the source mass, but have difficulties with background. Tracking experiments sacrifice source mass for extremely good background rejection. The seminal experiment of this design is the NEMO3 experiment. NEMO3 installs foils of the source isotope between tracking detectors and calorimeters. It can detect each electron as it is emitted from the foil, measure its energy and angular distribution and, thanks to a magnetic field, its charge. A display of an event detected in the NEMO3 is shown in Figure 8. The electrons are emitted from the source foil (pink line) at the same point, and then travel through the tracking chambers (red line) to the calorimeter (red boxes) where the electron energy is measured.

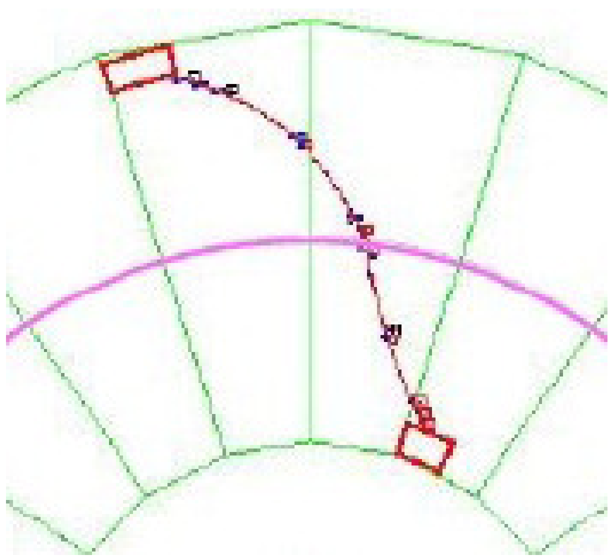

Figure 8: An event display from a background event in the Nemo3 detector.

The advantage of this detector is that is has phenomenal background rejection. The disadvantage is that the foils only allow a small amount of the source material to be used, and the energy resolution of the electrons is worse than for the semiconductor experiments (15% for 1 MeV electrons). NEMO3 has been running since 2003. Using a source isotope of ^{100}Mo it has put an upper bound of the effective neutrino mass of

$$m_{ee} < 0.8 - 1.3eV$$

A successor experiment, called SuperNEMO, is being designed now. It uses the same technology, but with more source material. It expects a sensitivity of $m_{ee} < 0.07 - 0.12$ eV.

0.4.4 What you should know

- What neutrinoless double beta decay is, and what the signal of such a decay looks like.
- The implications to neutrinos of a positive observation of neutrinoless double beta decay.
- General differences between the different types of neutrinoless double beta decay experiments.

Note that I won't ask detailed questions about specific experiments and their sensitivities.

0.4.5 Summary

Direct mass measurement using β decay is difficult. KATRIN will try to lower the bound to about 0.2 eV but it is clear that direct measurement experiments are reaching the limit of sensitivity. Neutrinoless double beta decay experiments are then the only hope of directly measuring the effecting neutrino mass.

The neutrinoless double beta decay industry is large and growing. The aim is to get the upper bound of the effective neutrino mass measurement below about 0.01 eV (in order to determine the mass heirarchy) or make a measurement of the effective mass. Only one experiment has claimed a measurement, and this is treated (as it should be) with scepticism until another experiment has confirmed it.