HW2 p.

~~Problem 2:~~ Problem 1:

$$|K^{*0}\rangle = |\bar{s}d\rangle = |\tfrac{1}{2}\ {-\tfrac{1}{2}}\rangle$$

$$|K^{0}\rangle = |\bar{s}d\rangle = |\tfrac{1}{2}\ {-\tfrac{1}{2}}\rangle$$

$$|\pi^{0}\rangle = |1\ 0\rangle \qquad |K^{-}\rangle = |s\bar{u}\rangle$$

$$|\pi^{+}\rangle = |1\ 1\rangle$$

$$|K^{0}\pi^{0}\rangle = |\tfrac{1}{2}\ {-\tfrac{1}{2}}\rangle|1\ 0\rangle = \sqrt{\tfrac{1}{3}}\,|\tfrac{1}{2}\ {-\tfrac{1}{2}}\rangle + \sqrt{\tfrac{2}{3}}\,|\tfrac{3}{2}\ {-\tfrac{1}{2}}\rangle$$

$$\mathcal{B}\quad \langle K^{*0}|K^{0}\pi^{0}\rangle = \sqrt{\tfrac{1}{3}}\,\langle \tfrac{1}{2}\|\tfrac{1}{2}\rangle$$

$$|K^{-}\pi^{+}\rangle = |\tfrac{1}{2}\ \tfrac{-1}{2}\rangle|1\ 1\rangle = \sqrt{\tfrac{2}{3}}\,|\tfrac{1}{2}\ {+\tfrac{1}{2}}\rangle + \sqrt{\tfrac{1}{3}}\,|\tfrac{3}{2}\ {+\tfrac{1}{2}}\rangle$$

$$\mathcal{B}\quad \langle K^{*0}|K^{-}\pi^{+}\rangle = \sqrt{\tfrac{2}{3}}\,\langle \tfrac{1}{2}\|\tfrac{1}{2}\rangle$$

$$\frac{Br(K^{*0}\to K^{-}\pi^{+})}{Br(K^{*0}\to K^{0}\pi^{0})} = \frac{\frac{2}{3}}{1/3} = 2$$

HW2 p.

Problem 2: Parity of Charged Pion

$\pi^- d \to n\,n$ is a strong interaction process $\Rightarrow$ parity is conserved.

Initial state angular momentum: $J = 1$

Final state: $J = L + S = 1$

deuterium has $S=1$
π has $S=0$
Capture takes place in $L=0$

let's look at the $|nn\rangle$ state under interchange.

Quantum numbers			Spin	space	total	allowed
J	L	S				
1	2	1	+	+	+	NO
1	1	1	+	−	−	yes
1	1	0	−	−	+	NO
1	0	1	+	+	+	NO

only possibility $\Rightarrow$ (row $J=1$, $L=1$, $S=1$)

Parity of final state $= (-1)^L (-1)^2 = (-1)^L = -1$

Parity of initial state $= P_{\pi^-} \cdot (-1)^L \cdot P_d = P_\pi \cdot 1 \cdot 1 = P_\pi$

$$\Rightarrow \boxed{P_\pi = -1}$$

Problem 3:

(a) $|\pi^0\rangle = |I, I_3\rangle = |1\,0\rangle$

$$|\pi^0\pi^0\rangle = |1\,0\rangle|1\,0\rangle = \sqrt{\frac{2}{3}}\,|2\,0\rangle - \sqrt{\frac{1}{3}}\,|0\,0\rangle$$

$$\Rightarrow \boxed{I = 2, 0 \text{ are allowed}}$$

(b) Wave function total must be symmetric because of Bose state.

$I = 0, 2$ are both symmetric

$\Rightarrow$ L must be even

(c) $J_{\rho^0} = 1$ $\quad \rho^0 \to \pi^0\pi^0$ is thus forbidden because $S_{\pi^0} = 0 \Rightarrow L = 1$ However 1 is odd thus not allowed as explained in (b).

HW2 p.

Problem 4:

(a) $|\pi^+\pi^-\pi^0\rangle = |11\rangle\,|1\,-1\rangle\,|10\rangle$

$|11\rangle|1\,-1\rangle = \frac{1}{\sqrt{6}}|20\rangle + \frac{1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{3}}|00\rangle$

$|20\rangle|10\rangle = \sqrt{\frac{3}{5}}|3,0\rangle - \sqrt{\frac{2}{5}}|10\rangle$

$|10\rangle|10\rangle = \sqrt{\frac{2}{3}}|20\rangle - \sqrt{\frac{1}{3}}|00\rangle$

$|00\rangle|10\rangle = |10\rangle$

$\Rightarrow$ $\boxed{I = 0, 1, 2, 3}$

(b) $|\pi^0\pi^0\pi^0\rangle = |10\rangle\,|10\rangle\,|10\rangle$

$|10\rangle|10\rangle = \sqrt{\frac{2}{3}}|20\rangle - \sqrt{\frac{1}{3}}|00\rangle$

$|10\rangle|00\rangle = |10\rangle$

$|20\rangle|10\rangle = \sqrt{\frac{3}{5}}|30\rangle - \sqrt{\frac{2}{5}}|10\rangle$

$\boxed{I = 1, 3}$

Problem 5:   Greiner QM 2 Symmetries
last part of Example 5.3

$$\sqrt{2}\,\langle n\pi^0 \| S | p\pi^- \rangle + \langle p\pi^- | S | p\pi^- \rangle = \langle p\pi^+ | S | p\pi^+ \rangle$$

This forms a triangle in the complex plane.

$$|n\pi^0\rangle = |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle |1\,0\rangle = \sqrt{\tfrac{2}{3}}\ |\tfrac{3}{2}\ -\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{3}}\ |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$$

$$|p\pi^-\rangle = |\tfrac{1}{2}\ \tfrac{1}{2}\rangle |1\ -1\rangle = \sqrt{\tfrac{1}{3}}\ |\tfrac{3}{2}\ -\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}}\ |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle$$

$$\langle n\pi^0 \| p\pi^- \rangle = \frac{\sqrt{2}}{3}\ \langle \tfrac{3}{2} \| \tfrac{3}{2} \rangle - \frac{\sqrt{2}}{3}\ \langle \tfrac{1}{2} \| \tfrac{1}{2} \rangle$$

$$\langle p\pi^- \| p\pi^- \rangle = \frac{1}{3}\ \langle \tfrac{3}{2} \| \tfrac{3}{2} \rangle + \frac{2}{3}\ \langle \tfrac{1}{2} \| \tfrac{1}{2} \rangle$$

$$\sqrt{2}\,\langle n\pi^0 | p\pi^- \rangle + \langle p\pi^- | p\pi^- \rangle = \left(\frac{2}{3} + \frac{1}{3}\right) \langle \tfrac{3}{2} \| \tfrac{3}{2} \rangle$$

$$= \langle \tfrac{3}{2} \| \tfrac{3}{2} \rangle$$

$$|p\pi^+\rangle = |\tfrac{1}{2}\ \tfrac{1}{2}\rangle |1\,1\rangle = |\tfrac{3}{2}\ \tfrac{3}{2}\rangle$$

$$\langle p\pi^+ | p\pi^+ \rangle = \langle \tfrac{3}{2} \| \tfrac{3}{2} \rangle \qquad \text{qed}$$

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Problem 6:

This is example 5.7 in Greiner QM2 Symmetries

$$p + d = \begin{cases} \pi^0 \ He^3 \\ \pi^+ \ H^3 \end{cases}$$

initial state: $|\tfrac{1}{2} \tfrac{1}{2}\rangle |00\rangle$

final states:

$$|11\rangle |\tfrac{1}{2} \ -\tfrac{1}{2}\rangle = |\pi^+ H^3\rangle$$

$$|10\rangle |\tfrac{1}{2} \ \tfrac{1}{2}\rangle = |\pi^0 He^3\rangle$$

$$R = \frac{\sigma(pd \to \pi^+ He^3)}{\sigma(pd \to \pi^0 H^3)} = \frac{|\langle 1 \tfrac{1}{2} \tfrac{1}{2} \| 1 \ -\tfrac{1}{2} \ \tfrac{1}{2}\rangle|^2}{|\langle 1 \tfrac{1}{2} \tfrac{1}{2} \| 1 \ 0 \ \tfrac{1}{2} \ \tfrac{1}{2}\rangle|^2} = \frac{2/3}{1/3}$$

$$R = 2$$