Spring Quarter 2015
UCSD Physics 214
Homework 1

Note, questions 1 and 3 are probably the most important of these questions! Q3 is meant to teach you the kinematics of hadron-hadron collisions. It's a must know item if you are working on the LHC. Question 2 is a simple exercise in relativistic kinematics that you should do once in your life. Question 1 introduces you to the CMS detector.

1. (40 points) This questions is meant to have you explore the CMS physics TDR volume 1 a bit. I will refer to information in the printed copy. However, there's also an online copy at https://cmsdoc.cern.ch/cms/cpt/tdr/ . You can find the material budget of the tracking system versus η in Figure 6.1 on p.226. The basic parameters of the tracking system are summarized in 1.5.5. Efficiencies are given in Figure 1.12 on p.23. The ECAL performance is summarized in 1.5.3.
    a. Figure 1.5 shows the tracking resolution for muons using the tracker only for two different ranges in η. In class we discussed a simplified model for the tracking resolution. Take that model and plot the corresponding resolution vs momentum. As inputs assume a point resolution of 40 microns for all layers. Plot the resolution for η=0 and η=1.6, and determine the momentum at which intrinsic resolution and multiple scattering are equal. How does this compare with the typical momentum for muons from Z decay?
    b. Given the material budget of the tracker, what fraction of pions undergo hadronic interaction before they reach the ECAL? Calculate this for η=0 and η=1.6 . Compare this with the efficiency for pions given in Figure 1.12.
    c. What's the minimum momentum required to reach the outer most silicon layer at η=0?
    d. For a 50GeV electron, compare the expected loss of energy due to radiation with the intrinsic resolution of the ECAL at that energy. Do so for η=0 and η=1.6. At what energy is the intrinsic resolution roughly equal to the average energy radiated for these two values of η. To be fair, the electron reconstruction algorithm in CMS accounts for this by allowing a much larger shower size in phi than η. A significant fraction of the radiated photon energy is thus measured in the ECAL, and attributed correctly to the electron track.
    e. Electrons from pair production are a significant source of "fake" electron backgrounds. The tracking algorithm in CMS used to requires hits in all three layers of the pixel detector. What percentage of high energy photons at our two favorite η points (0 and 1.6) produce conversion electrons that might fake electrons from the interaction region? (Hint: Assume that 1/3 of the total material in the pixel detector is the relevant material budget for this question.)

2. (20 points) A particle of mass M decays at rest into particles of mass m1 and m2 with m1 > m2.
    a. Calculate the energy, kinetic energy, and momentum of the two particles. Express the momentum in terms of μ=m1+m2 and δm=m1-m2. Which particle has higher energy? Which particle has the higher kinetic energy?
    b. Compute these energies and momenta for the decay D*+ -> D0 π^+ .
    c. Suppose the original particle has momentum q in the LAB. Find the momentum of the two decay products in the LAB as a function of the decay angle, cos(theta) in the mother particles restframe with respect to the flight path of the original particle. For the D*+ -> D0 π^+ plot the LAB momenta vs cos(theta) for q=1GeV and q=10GeV.
3. (40 points) Rapidity (y) is a kinematical variable which is useful in describing hadron-hadron collisions, e.g. at the LHC. Consider pp collisions, rapidity is defined as: $y = \frac{1}{2}\ln\frac{E + p_L}{E - p_L}$ where E is the energy of the particle and p_L is the longitudinal momentum, i.e. the z-component of the momentum, i.e. the component of the momentum along the incoming proton direction. Let us now investigate some properties of y.
    a. We will show later in the quarter that the cross section for particle production is proportional to the phase space factor d^3p/E, where p and E are the three momentum and energy of the particle. Show that: $\frac{d^3p}{E} = \frac{1}{2}dp_T^2 dy d\phi$ where phi is the angle in the plane orthogonal to the incoming beam. Since the cross section for particle production depends mostly on p_T and only very weakly on y, particle production is expected to be uniform in rapidity. This is referred to as the "rapidity plateau".
    b. Show that rapidity differences between two particles are independent of boost along the z-direction. This is the reason why rapidity is the preferred coordinate in hadron colliders. After all the boost of the parton-parton collisions is unknown given that the momentum fraction of the partons inside the colliding protons is not known on an event by event basis.
    c. Show that for a particle of mass m in the LAB frame: $-\frac{1}{2}\ln\frac{s}{m^2} < y < \frac{1}{2}\ln\frac{s}{m^2}$ where s is the square of the COM energy of the pp collision while m is the mass of the particle. This relationship defines the end of the rapidity plateau. Use this relationship to calculate the rapidity range for Z production at 2TeV and 14TeV, and compare this with the corresponding range for a hypothetical 500GeV particle at the two colliders.
    d. Now consider a gluon from a high transverse-momentum collision. The gluon will fragment into a collimated spray of hadrons, called jets. These hadrons have small momenta transverse to the gluon direction. Let q_L and q_T be the typical longitudinal and transverse momenta of the hadrons with respect to the gluon direction. Take $q_T \ll q_L$, and $m \ll q_L$, with m being the typical mass of the hadrons.

Show that jets are approximately circular in η-ϕ space (η=ln(cot(theta/2))), i.e. that the typical spreads in these two coordinates is roughly equal given a gluon direction and q_T, q_L.
Also show that the η-ϕ size of the jet is roughly independent of the gluon rapidity.
For these reasons, jet momenta are often measured by summing up the momenta of all hadrons in a fixed cone in η-ϕ.

In addition, I am offering you some extra credit questions:

1. (20 points extra) These are some introductory questions to get you acquainted with the Particle Data Book, as well as provide you with some sense of scale.
   a. Look up the masses of the W and Z boson, and compare them with the mass of the proton. Estimate the approximate range of the forces mediated by W,Z.
   b. Look up the lifetimes of μ^+, τ^+, K^+, K_s , K_L, π^+, π^0, ρ^0, D^+, B^+, J/ψ. From the values of the lifetime guess the interactions responsible for these decays. Calculate the average distance traveled by these particles when they are produced with a momentum of 10GeV.
   c. Look at the possible decay modes of the muon and the tau. Why are the decay modes of these two leptons so different?
   d. At the top of the atmosphere, high energy cosmic rays produce an equal number of π^+ and π^-which then decay. Their decay products can then decay further. What is the ratio of the number of electron to muon neutrinos that reach the earth. Assume that all unstable particles decay before they reach the earth, which is not a correct assumption.
   e. Look at the lifetime of the D+ and D0 meson. Both decay via weak interaction. Can you explain the lifetime difference semi-quantitatively? Hints:
      i. They decay via W emission, either internal or external.
      ii. All mesons are colorless, whereas the W is color neutral. Color matching is thus required when combining mesons.
2. (20 points extra) One of the most striking signatures for some parts of the supersymmetry parameter space is the decay χ_2^0 to χ_1^0 e^+e^- via an intermediate resonance. In this decay, the di-electron mass follows a triangular distribution. Show that this is a general characteristic of any decay A to B + C followed by C to D + E. If B,D can be considered massless, then the B-D invariant mass will display this triangular form, assuming there is no angular momentum that matters in either of the two decays. I challenge you to either show this explicitly, by deriving the shape in the B-D invariant mass distribution, or to write a simple Monte Carlo generation to prove it "experimentally".