Physics 1C

Lecture 28B

Essential Concept of last lecture

- Light is emitted and absorbed in form of quantas of energy that are multiple of a minimal energy quantum.
  - Emission: e.g. black body radiation
  - Absorption: e.g. photoelectric effect
- This minimal quantum of energy depends of the frequency of the light as follows:

$$E_0 = hf \Rightarrow E_n = nhf$$

- $h = 6.626 \times 10^{-34}$ Joule x seconds is a constant of nature called Planck's constant.

The Compton Effect

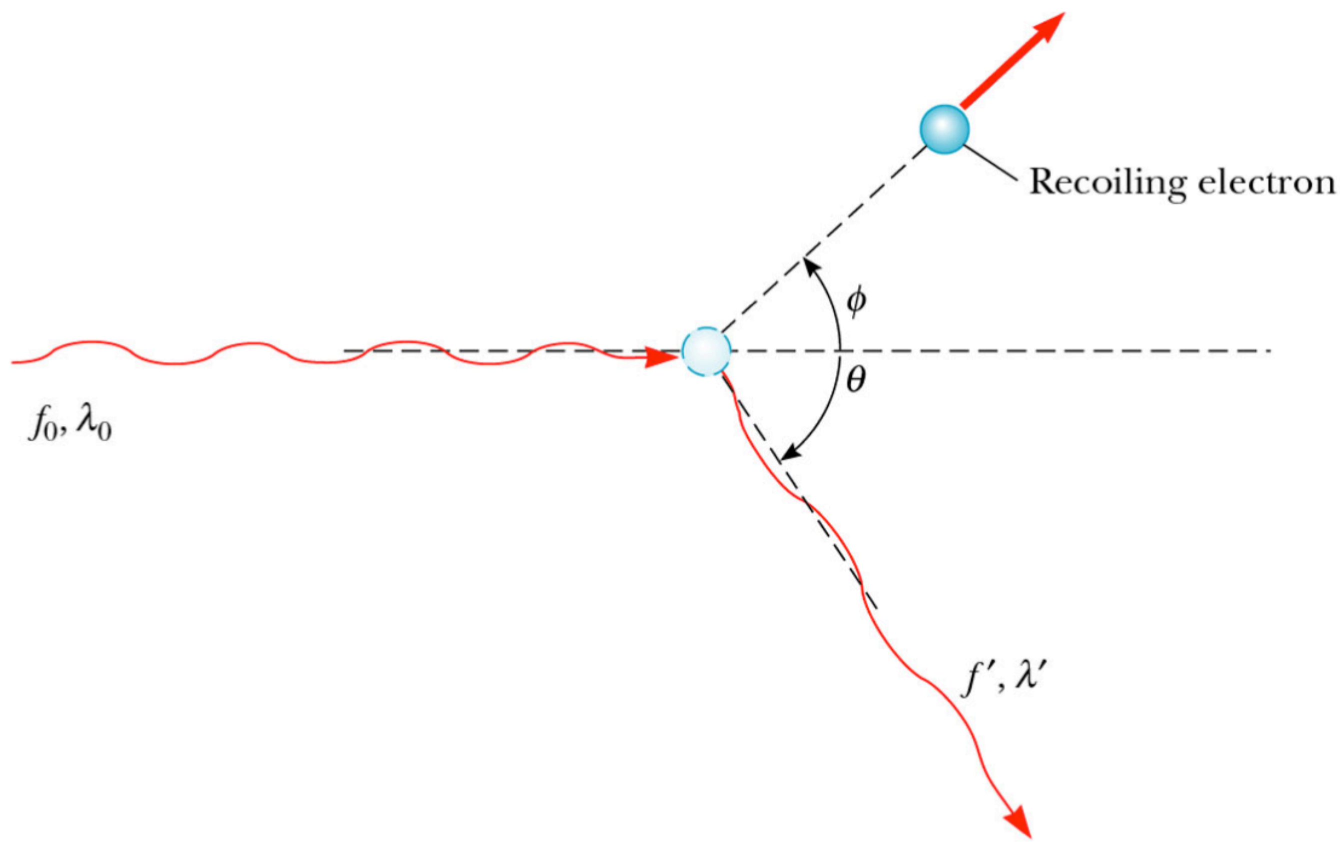
- In 1923, Arthur Compton (U of Chicago) directed a beam of x-rays toward a block of graphite
- He detected the scattered x-rays had a slightly longer wavelength than the incident x-rays.
- This means the scattered photons had less energy than the incident photons.
- The amount of energy the scattered photons lost depended on the angle at which the x-rays were scattered.
- This change in wavelength is called the **Compton shift** (1927 Nobel Prize).

The Compton Effect

- To calculate the shift in wavelength, Compton assumed that the photons act like other particles in collisions.
- In the collisions, energy, *hf,* and momentum, *hf/c*, were conserved.
- The energy of the incoming photon was:

$$E_o = hf_o = h\frac{c}{\lambda_o}$$

- After it collides it scatters by an angle θ.



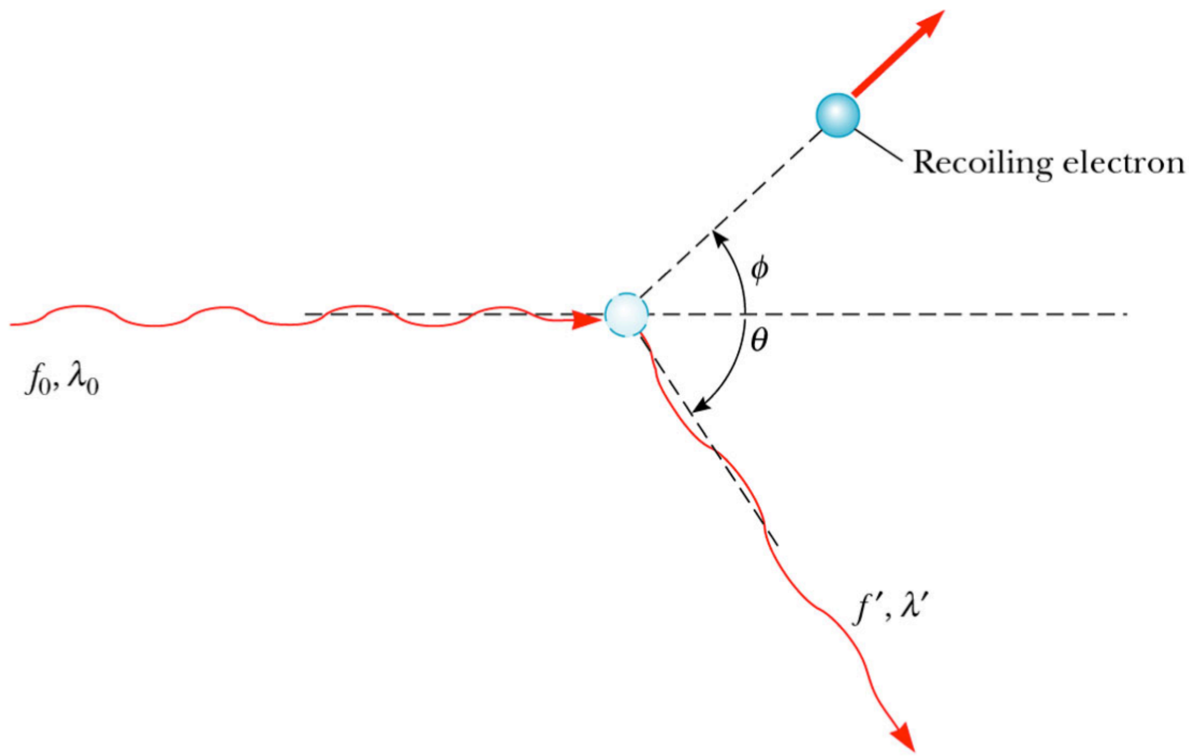
The Compton Effect

- After the collision the photon has an energy:

$$E_f = hf = h\frac{c}{\lambda}$$

- The wavelength shift becomes: $\lambda' - \lambda_o = \frac{h}{m_e c}\left(1 - \cos\theta\right)$
- where the **Compton wavelength** for the electron is a constant:

$$\frac{h}{m_e c} = 0.00243\ nm$$



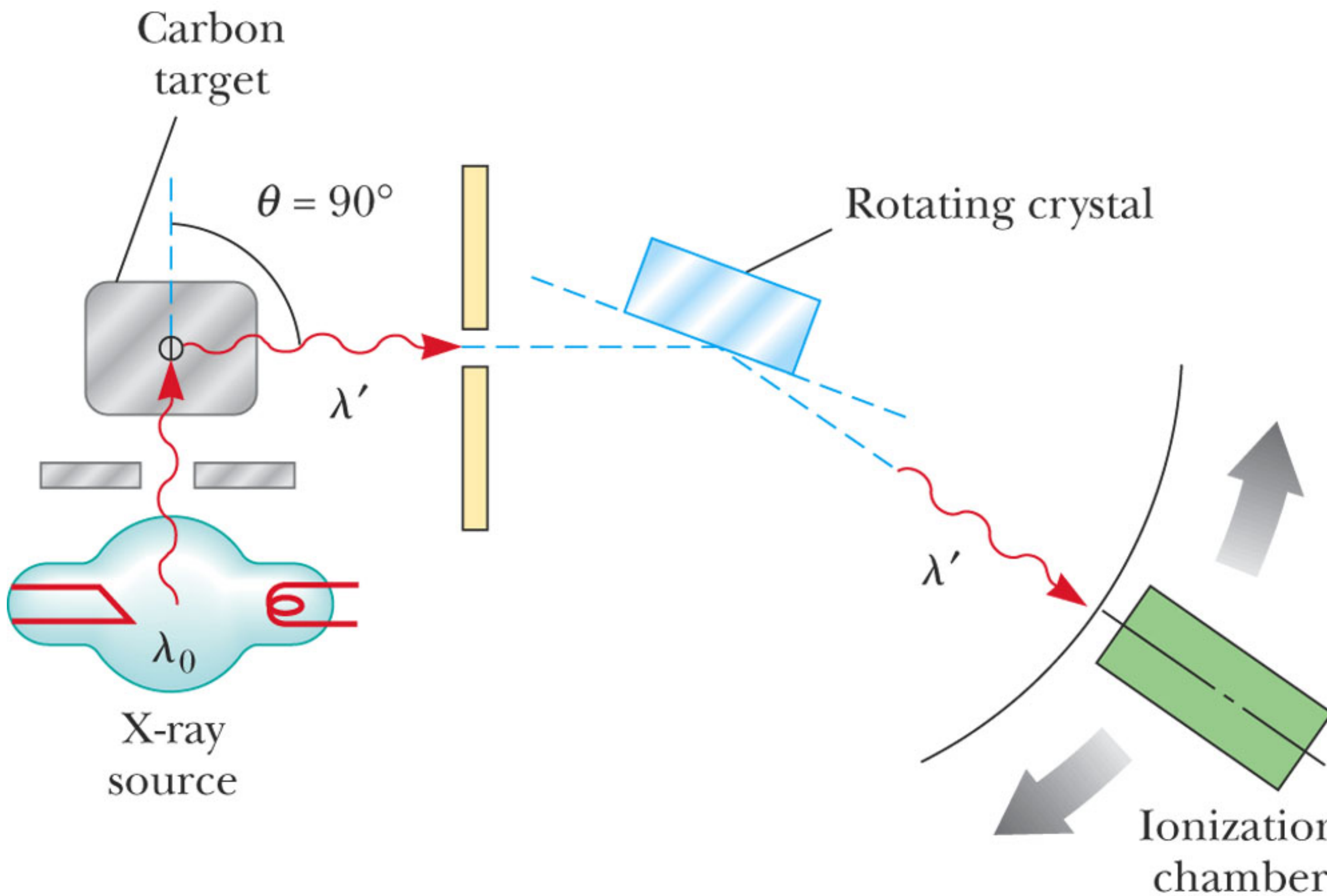
The Compton Effect

- Schematic diagram of Compton's apparatus:

- An x-ray tube produces radiation of

  λ_o = 0.0707nm that strikes a carbon target.

- Angle θ is varied by moving the x-ray source.



- The x-ray spectrometer includes a crystal that reflects x-rays and an ionization chamber that measures *I*.

The Compton Effect

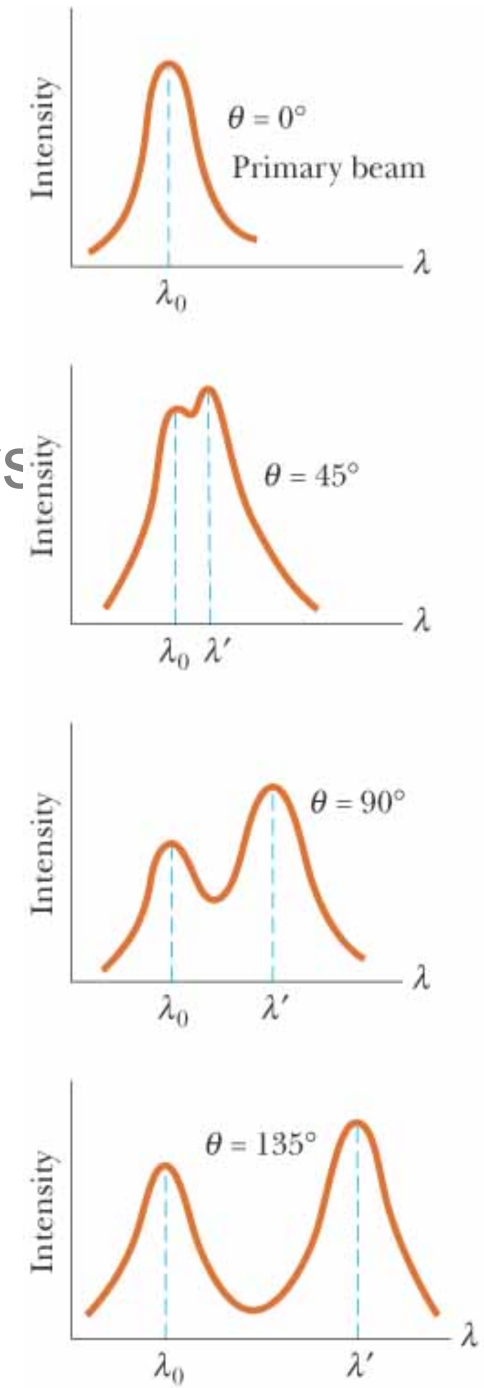
The wavelength of the scattered x-rays can be determined from the angle at which they were reflected from the crystal with maximum intensity (x-ray diffraction).

The graphs show the spectra of scattered x-rays for various angles θ.

The shifted peak at λ' is caused by the scattering of free electrons in the target:

$$\lambda' - \lambda_o = \frac{h}{m_e c}(1 - \cos\theta)$$

The unshifted wavelength, λ_o, is due to x-rays scattered from the electrons that are tightly bound to the target atoms.



Wave Particle Duality

- Experiments such as the photoelectric effect, the Compton effect, and double-slit interference helped to describe light's true nature: a wave and a particle.
- Light has a dual nature, exhibiting both wave and particle characteristics.
- This means a photon can have energy given by:

$$E = hf = h\frac{c}{\lambda}$$

- And that photons can have a momentum given by:

$$p = \frac{E}{c} = \frac{1}{c}\left(h\frac{c}{\lambda}\right) = \frac{h}{\lambda}$$

Wave Particle Duality

- Then in 1923, Louis de Broglie (Sorbonne, Paris) proposed a very interesting hypothesis:
- Because photons have wave and particle characteristics, perhaps all forms of matter have both properties.
- de Broglie proposed that subatomic particles, like protons and electrons, will have wavelengths just like photons.
- de Broglie suggested that a particle of mass *m* and velocity *v* would have a wavelength of:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

- ← This is known as the **de Broglie wavelength** of a particle.

Wave Particle Duality

- de Broglie also postulated that **matter waves** have **frequencies** that can be found as:

$$f = \frac{E}{h}$$

- But de Broglie <u>only hypothesized</u> about matter waves
  - What kind of experiment could we design to demonstrate the wave nature of particles (like electrons)?
- We could shoot electrons through a double slit apparatus and observe if there is a resulting interference pattern (similar to the pattern we observed for light).

Wave Particle Duality

- **Davisson and Germer** (US) performed this experiment in 1927.
- They scattered low energy electrons from a nickel target in a vacuum.
- From this experiment they found (by accident!) interference patterns and calculated a wavelength for the electron.
- This wavelength agreed with the theoretical de Broglie wavelength.
- This confirmed the wave nature of electrons.
- This was the first experimental confirmation of the de Broglie hypothesis.

Examples

A. Calculate the de Broglie wavelength of an electron (m_e = 9.11×10^{-31} kg) moving with a speed of 1.00×10^{7} m/s.

Answer: $\lambda = h/(m_e v) = 0.0727$ nm

This λ is close to the characteristic wavelength of x-rays.

It is also on the order of the spacing of atoms in the sodium chloride lattice.

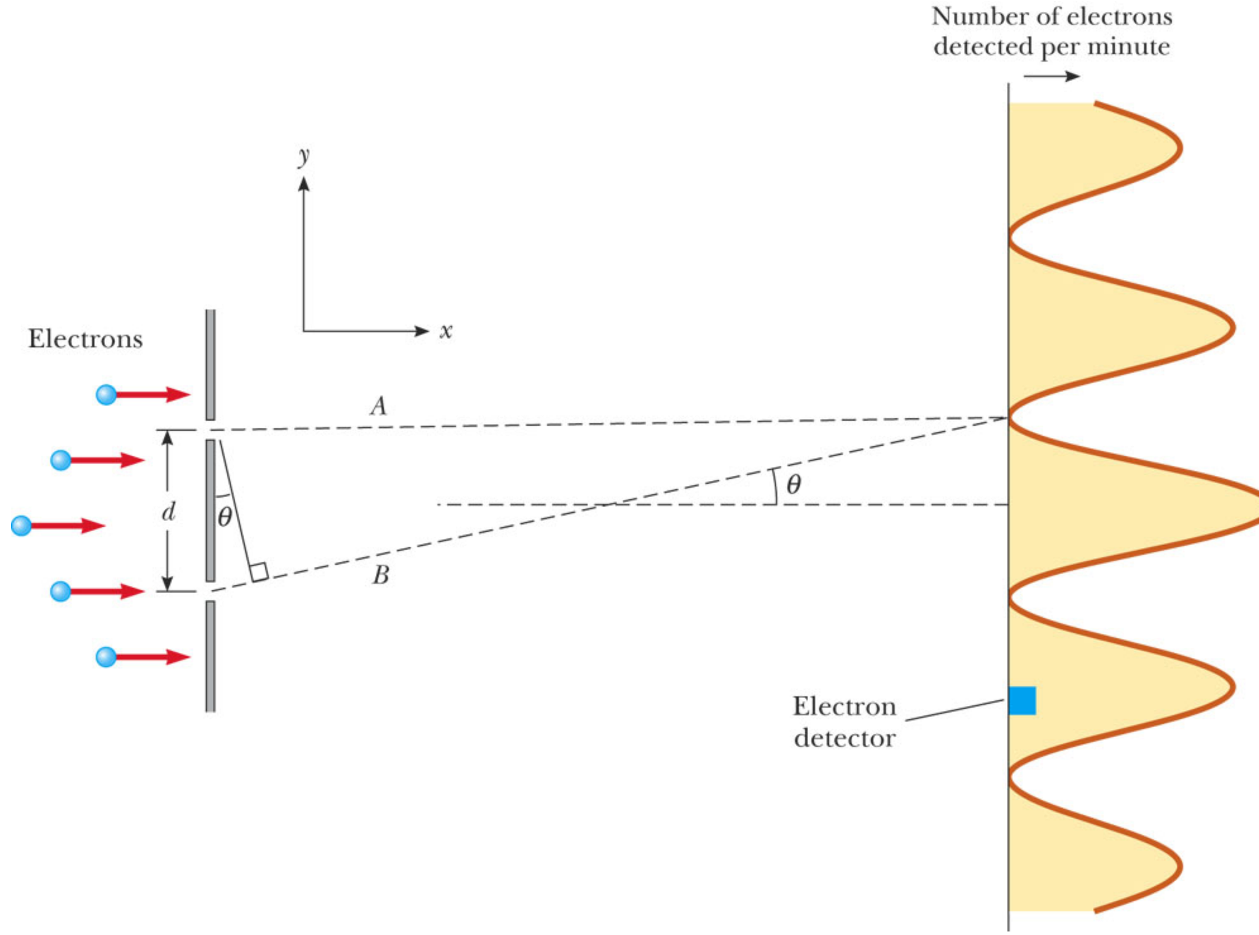
B. Calculate the de Broglie wavelength for a rock of mass 50.0 g thrown with a speed of 40 m/s.

Answer: 3.31×10^{-34} m

This wavelength is much smaller than the rock. Thus, the wave properties (e.g. diffraction effects) of large-scale objects cannot be observed.

Electron Diffraction

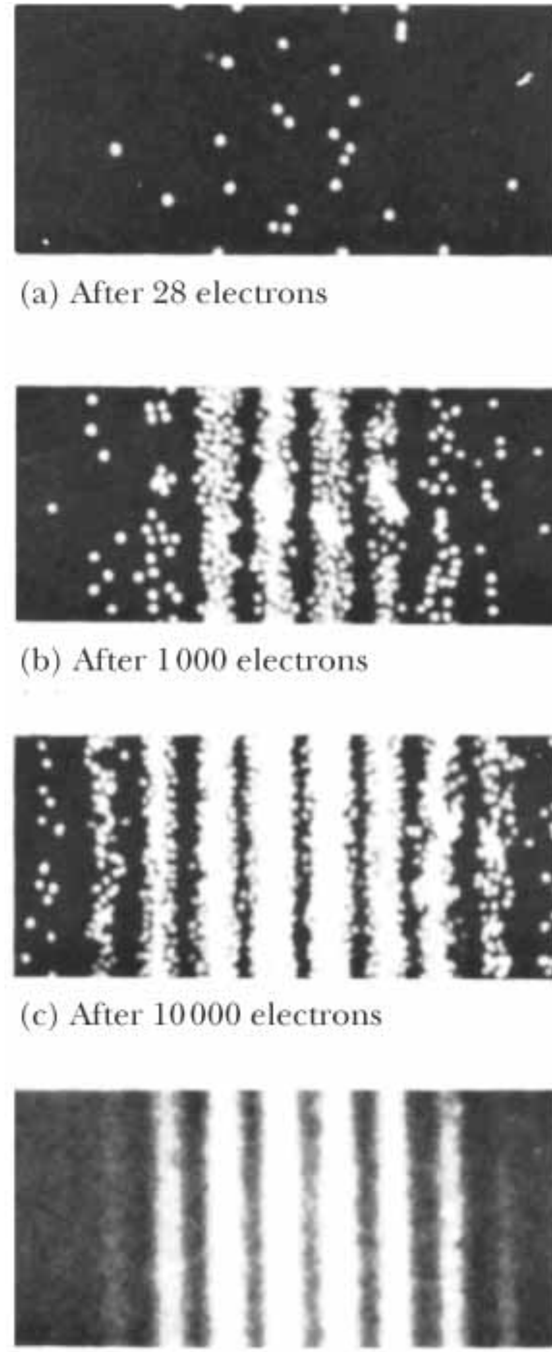
- Consider a parallel beam of monoenergetic electrons that is incident on a double slit.
- Assume that the slit width are small compared to the electron wavelength.
- An electron detector is positioned at a distance *L*>>*d*.



- A typical wave interference pattern for the electron counts per minute appears.

Electron Diffraction

- It is clear that electrons are interfering, which is a distinct wave-like behavior.
- If the experiment is carried out at lower electron beam intensities, the interference pattern is still observed if the exposure is sufficiently long.
- As in Chapter 27, we can use the waves in interference model to find the angular separation θ between the central maximum and the first minimum: $d \sin \theta = \lambda/2$.
- Because for electrons $\lambda = h/p_x$ we get for small θ: $\sin \theta \approx \theta = h/(2p_x d)$



(a) After 28 electrons

(b) After 1000 electrons

(c) After 10000 electrons

(d) Two-slit electron pattern

Electron Diffraction

- The electrons are detected as particles at a localized spot at some instance in time, but the probability of arrival at that spot is determined by finding the intensity of two interfering waves.
- We conclude that an electron interacts with both slits simultaneously.
- It is impossible to determine which slit the electron goes through.
- We can only say that the electron passes through both slits!
- If we attempt to experimentally determine which slit the electron goes through, the interference pattern is destroyed.
- The same argument applies to photons.

The Uncertainty Principle

- When measurements are made, the experimenter is always faced with experimental uncertainties in the measurements.
- In classical mechanics, there can be measurements with arbitrarily small uncertainties (no limit).
- Yet quantum mechanics predicts that a barrier to measurements with ultimately small uncertainties does exist.
- When taking the measurement of position, x, the uncertainty of the measurement is given by Δx.
- When taking the measurement of momentum, p_x, the uncertainty of the measurement is given by Δp_x.

The Uncertainty Principle

- In 1927, Werner Heisenberg unveiled his uncertainty principle:
- The product of the uncertainty of the position of a particle (Δx) and the uncertainty of the linear momentum of the particle (Δp_x) can never be smaller than $h/4\pi$.
- Mathematically, this becomes: $$\Delta x \Delta p_x \geq \frac{h}{4\pi}$$

- Basically, the uncertainty principle states that **it is physically impossible to simultaneously measure the exact position and the exact linear momentum of a particle**.

The Uncertainty Principle

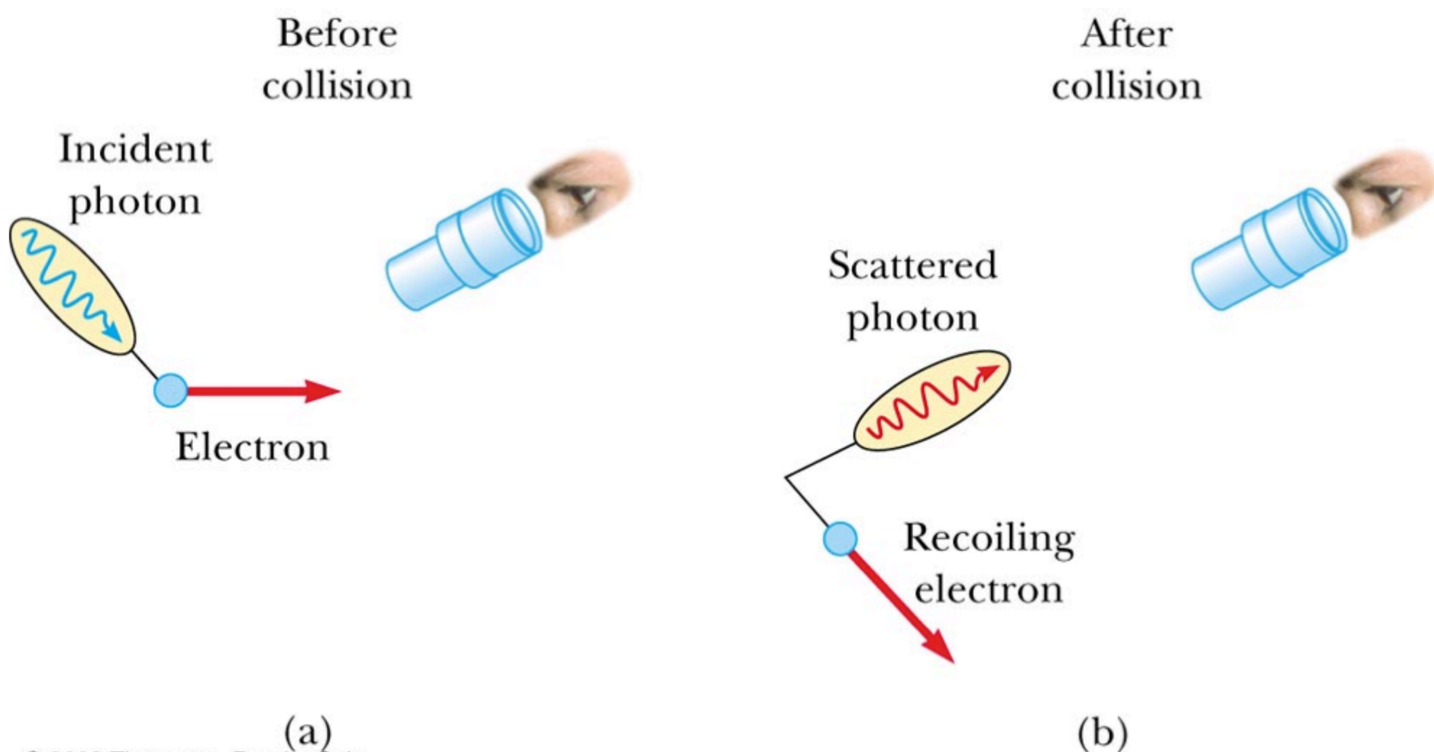
- The uncertainty principle can be extended to deal with energy uncertainties as well.
- In this case, position and linear momentum are replaced with energy and time such that:

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

- In this form, the uncertainty principle states that it is physically impossible to measure the exact energy of a particle during a finite period of time.

The Uncertainty Principle

- One way to consider the implications of the uncertainty principle is to think about how to measure the position and linear momentum of an electron with a very powerful microscope.
- In order to locate the electron accurately, at least one photon must bounce off of it.
- During this interaction, momentum is transferred from the photon to the electron.



The Uncertainty Principle

- Therefore, the light that allows you to accurately locate the electron changes the momentum of the electron (the maximum change is $\Delta p_x = h/\lambda$).
- To minimize the momentum change, we could use longer wavelength photons.
- But because the photon also has wave properties, we can determine the electron position only within one wavelength of the photon, $\Delta x = \lambda$.
- Consequently, the position and the momentum of the electron cannot both be known precisely at the same time: $\Delta p_x \Delta x \geq h$.
- Apart from the numerical factor $1/4\pi$, this formula is the same as Heisenberg's more precise result.

For Next Time (FNT)

- Finish reading Chapter 28
- Start working on the homework for Chapter 28