Physics 1C

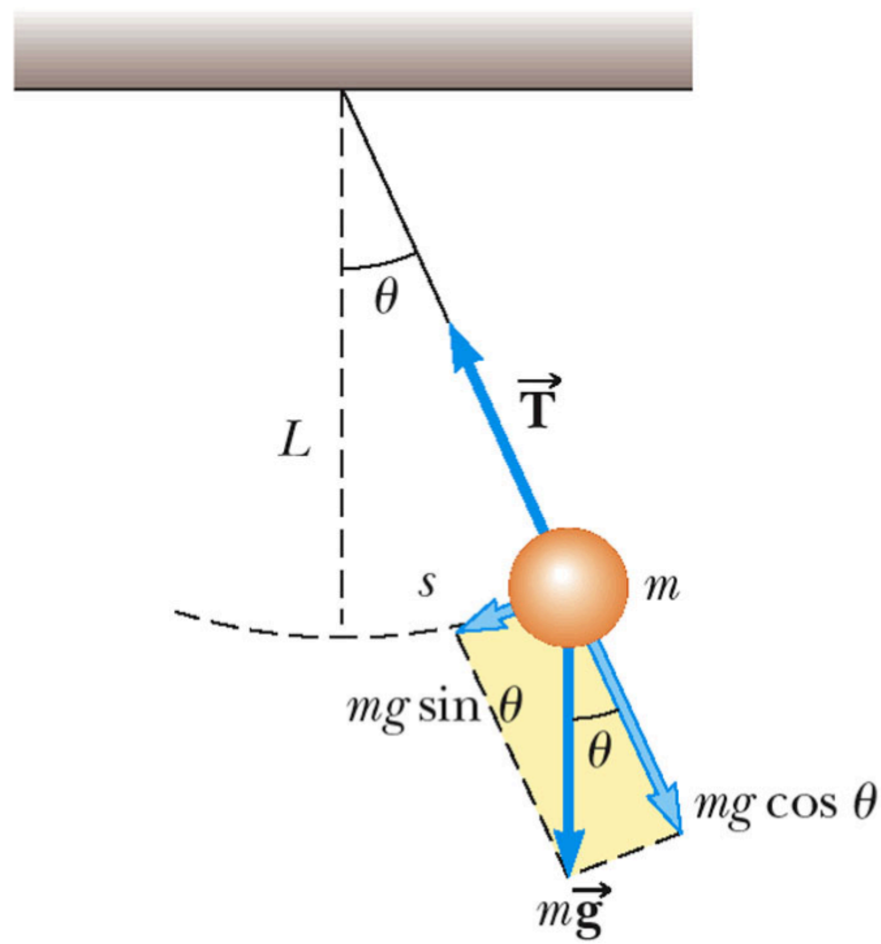
Lecture 12C

Simple Pendulum

The simple pendulum is another example of simple harmonic motion.

Making a quick force diagram of the situation, we find:

- The tension in the string cancels out with a component of the gravitational force ($mg\cos\theta$).
- This leaves only the perpendicular component causing acceleration.



Simple Pendulum

Looking back at Hooke's Law:

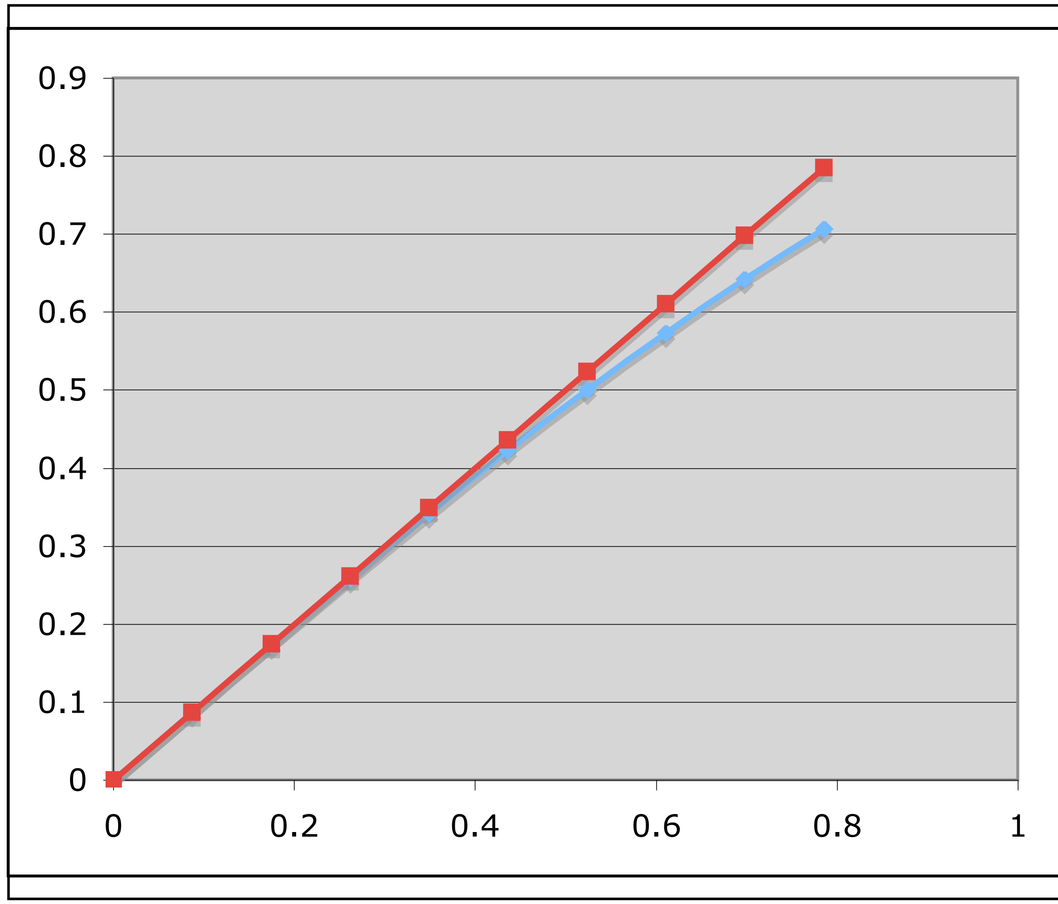
$$\sum \vec{F} = \vec{F}_{restoring} = -(\text{constant})(\text{displacement})$$

- In this case we have:

$$F_{g,x} = -mg\sin\theta$$

- It doesn't seem to follow the pattern for Hooke's Law.

- But at small angles (θ<30°), we can say that $\theta \cong \sin\theta$ (if θ is in radians)



Simple Pendulum

In the tangential direction:

$$F_t = ma_t \rightarrow -mg\sin\theta = m\frac{d^2s}{dt^2}$$

The length, L, of the pendulum is constant and for small values of θ:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

$$\tan\theta = \frac{s}{L} \Rightarrow \frac{d^2s}{dt^2} = L\frac{d^2\theta}{dt^2}$$

This confirms the form of the motion is SHM!

General solution:

$$\theta = \theta_{max}\cos(\omega t + \phi)$$

Simple Pendulum

The angular frequency is:

$$\omega = \sqrt{\frac{g}{L}}$$

The period is:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

The period and frequency depend only on the length of the string and the acceleration due to gravity

The period is independent of the mass

Pendulum Period

- All simple pendula that are of equal length and are at the same location (same *g*) oscillate with the same period

- Please note that the period of a pendulum depends on different variables than the period of a mass on a spring

Pendulum

- **Example**
- A 300g mass on a 30cm long string oscillates as a pendulum. It has a speed of 0.25m/s as it passes through the lowest point. What maximum angle does the pendulum reach? (Assume the angle remains small enough for the motion to be SHM.)

- **Answer**
- First, you must define a coordinate system.
- The lowest possible point can be $y=0$, let's define up as $+y$.

Pendulum

Answer

Next, let's turn to conservation of energy.

Choose the initial state to be when the mass is moving through its lowest point:

$$E_i = KE_i + PE_i = KE_{\max} + 0$$

$$E_i = \frac{1}{2} m v_{\max}^2$$

Choose the final state to be when the mass reaches its highest point:

$$E_f = KE_f + PE_f = 0 + PE_{\max}$$

$$E_f = mgh_{\max}$$

Pendulum

Answer

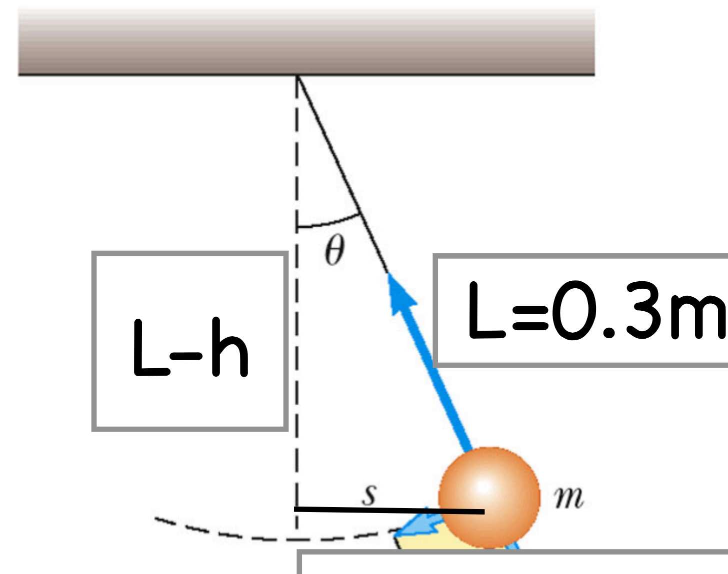
Energy is conserved, so: $E_i = E_f$

$$\frac{1}{2}mv_{max}^2 = mgh_{max}$$

$$h_{max} = \frac{v_{max}^2}{2g} = \frac{\left(0.25\,{}^{m}/_{s}\right)^2}{2\left(9.8\,{}^{N}/_{kg}\right)} = 3.19\text{mm}$$

What does this height represent?

The height it rises from the lowest to highest point.
The angle we want to solve will have a hypotenuse of 0.3m and an adjacent side of *L-h*.



$$\cos\theta = \frac{L-h}{L}$$

$$L - h = 0.3\text{m} - 0.00319\text{m} = 0.2968\text{m}$$

$$\theta = \cos^{-1}\left(\frac{0.2968\text{m}}{0.3\text{m}}\right) = 8.4°$$

Waves

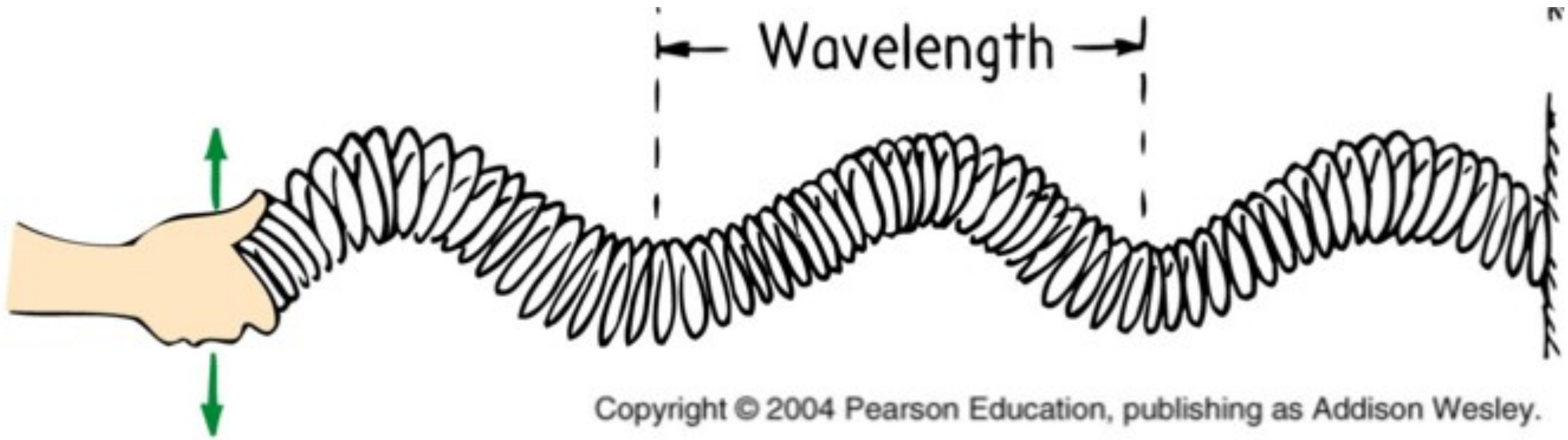
- **Wave motion:** transfer of disturbance through space without transfer of matter
- <u>**Mechanical waves**</u> are waves that disturb and propagate through a medium
- <u>Examples:</u> water waves, waves on a string, sound waves
- <u>**Electromagnetic**</u> waves are a special type of waves that do not require a medium to propagate
- <u>Examples</u>: light waves, radio waves

Waves

- A mechanical wave:
- Requires a medium that can be disturbed
- Requires some source of disturbance
- Requires some physical mechanism through which elements of the medium can influence one another
- Carries energy and momentum from one spatial location to another
- Can be sinusoidal or other shapes as well

Types of Waves

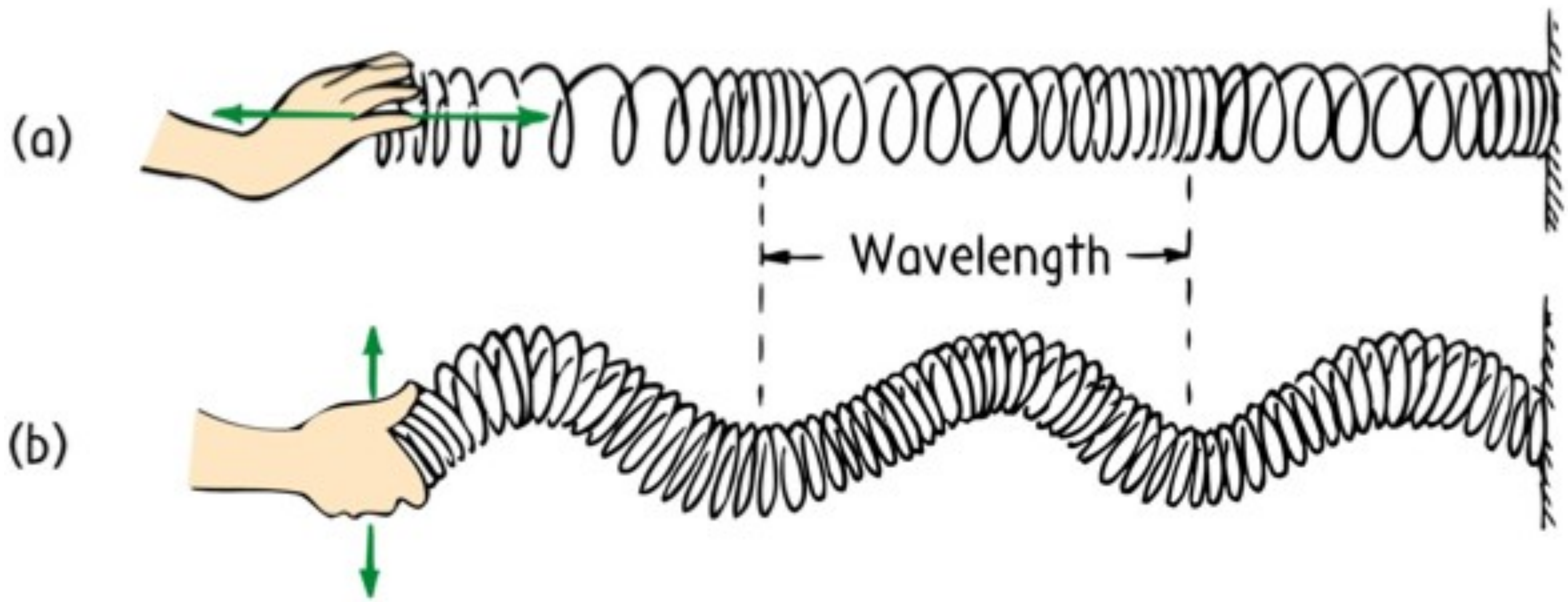
- **Transverse:**
- Displacement is perpendicular to the direction of wave propagation.



- Examples of transverse waves are:
- water waves, EM waves, waves on a string...

Types of Waves

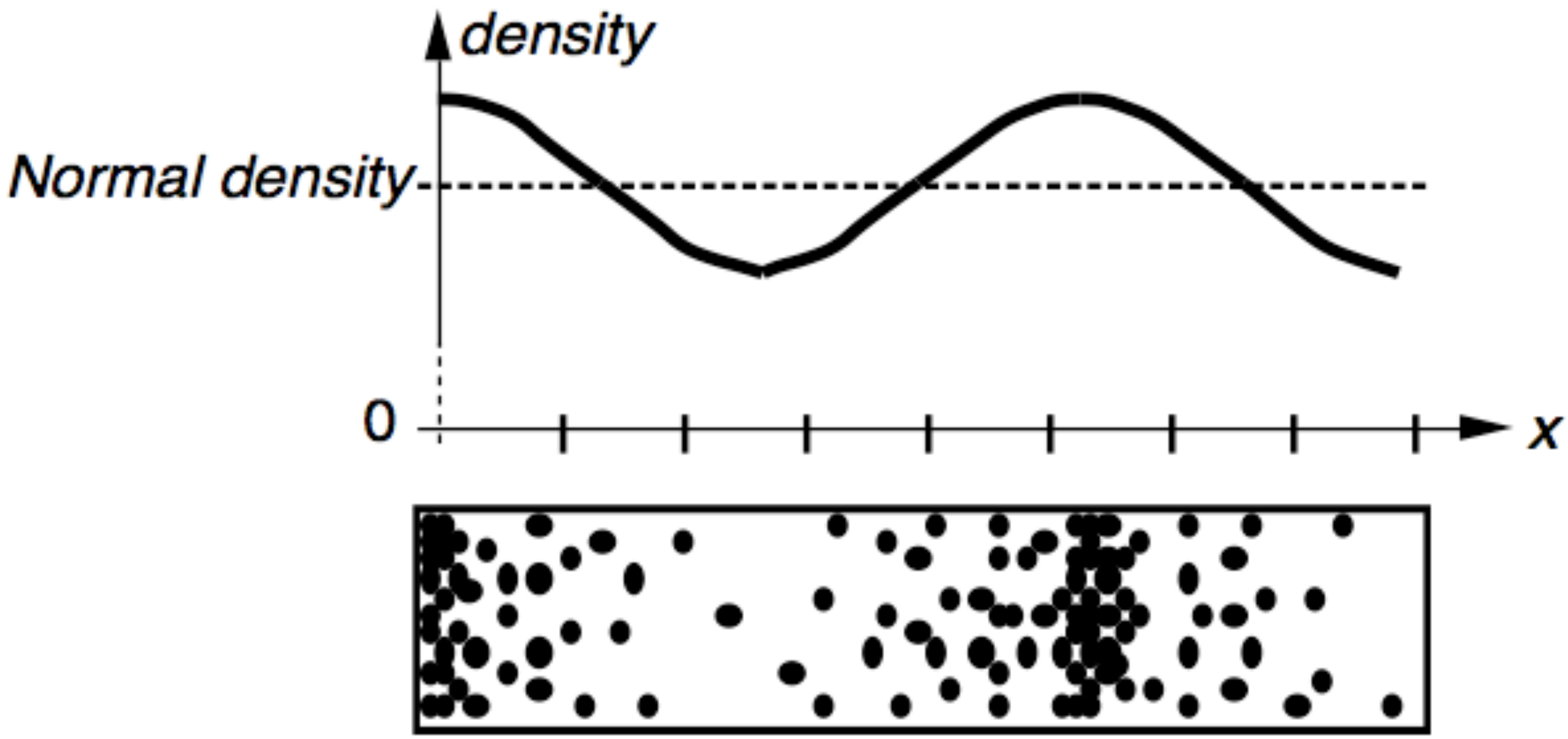
- **Longitudinal:**
- Displacement is in the direction of wave propagation.



- Examples of longitudinal waves are:
- sound waves, compression waves on a slinky...

Types of Waves

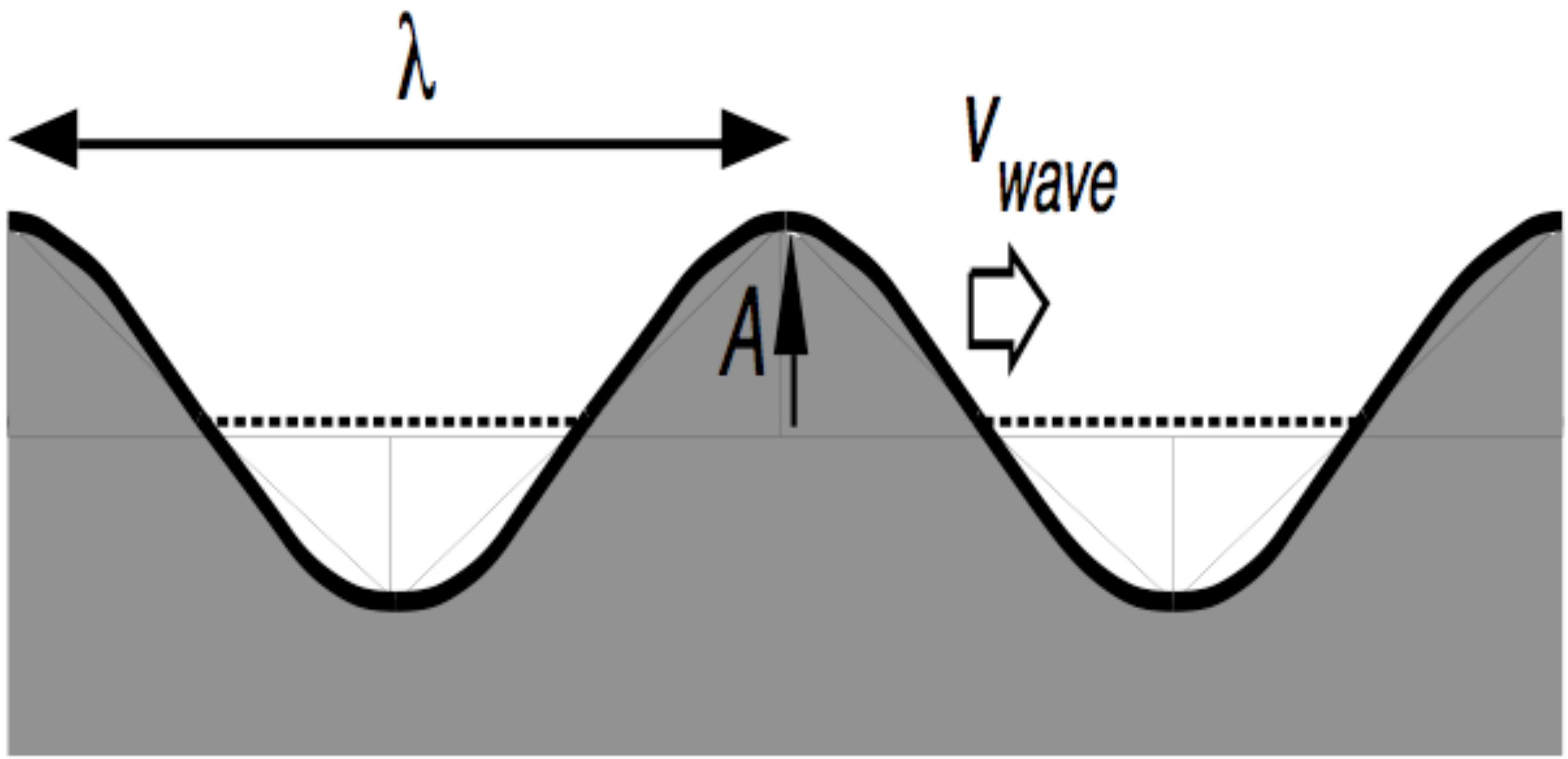
- Longitudinal waves are described the same way as transverse waves. It's just that the amplitude means something different.



- We can use the same equations and techniques to describe both types of waves.

Pictorial Description

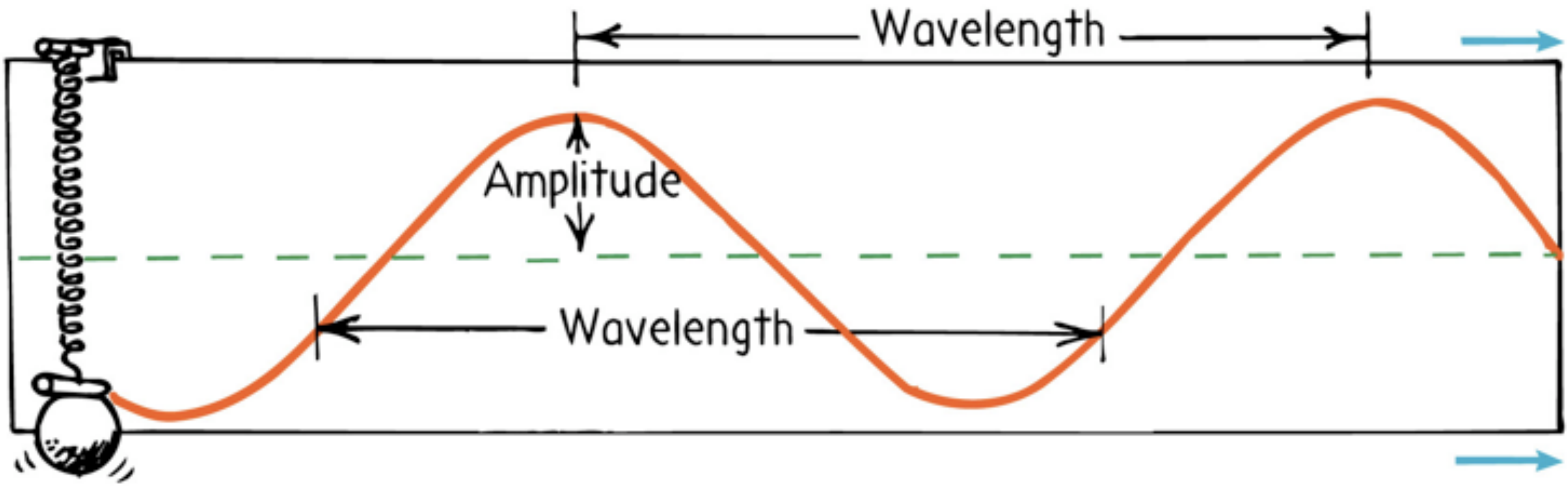
- We like to represent a traveling wave pictorially with a moving sine wave.



- Here the traveling wave is moving to the right at a certain velocity, v_{wave}.

Pictorial Description

- The highest points of the wave are known as **crests**. The lowest points of the wave are known are as **troughs**.

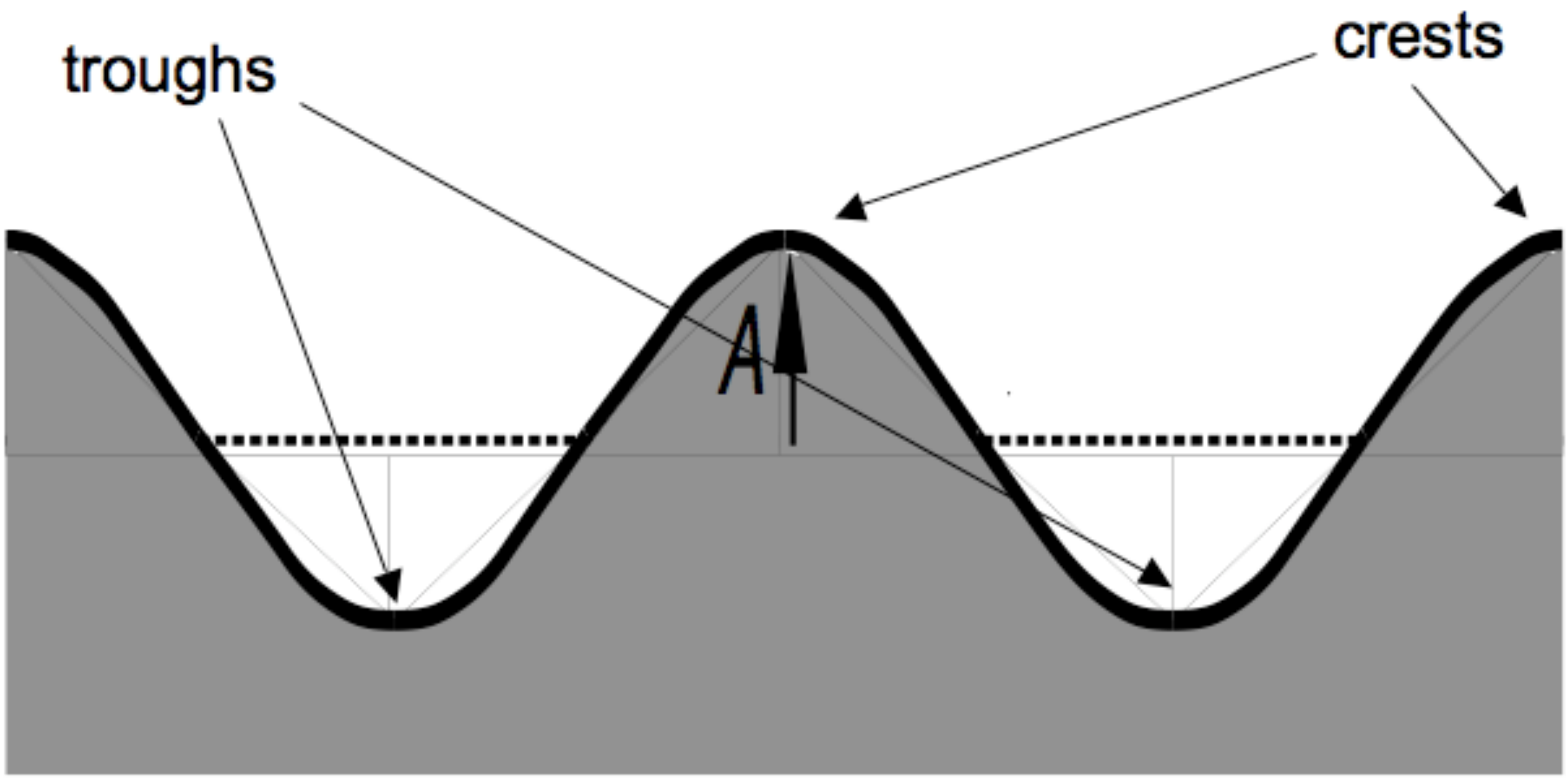


- The difference between equilibrium and crest (or trough) is called the **amplitude**, *A*.

- The distance from one crest to another (or trough to trough) is called the **wavelength**, λ.

Pictorial Description

- The time it takes between successive crests is called the **period**, *T*.
- If we were sitting on the middle crest, then the period would be how much time it took for the next crest to arrive.
- If we invert period, *T*, then we have frequency, *f*.



Speed of a Wave

- Very often you will be asked how fast the wave is traveling.
- From the variables that we just described, the speed of a wave is given by:

$$v_{wave} = \frac{\lambda}{T} = \lambda f$$

- It comes from the fact that speed equals distance over time.
- This is a very useful equation that we will use quite often.

Speed of a Wave

- For example, an ocean wave is traveling in one direction has a wavelength of 1.0m and a frequency of 1.25Hz. What is the speed (in m/s) of this ocean wave?

$$v_{wave} = \lambda f = 1.25 \,{}^{m}\!/_{s}$$

- Please note that the water is not actually moving at this speed, but the wave is propagating at this speed.

- If a rubber duckie were sitting on the water it would most likely just bob up and down.

For Next Time (FNT)

- Continue reading Chapter 13
- Start working on the homework for Chapter 13