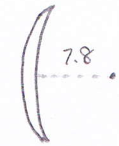
Practice Final Solution

1. $T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow 2 = 2\pi\sqrt{\frac{L}{9.8}} \Rightarrow L = 0.99\ (m)$ //



2. $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, $f = 444\ mm$, $n = 1.56$, $R_2 = 7.8$

$\frac{1}{444} = (1.56-1)\left(\frac{1}{R_1} - \frac{1}{7.8}\right) \Rightarrow \frac{1}{R_1} - \frac{1}{7.8} = 4.02\times10^{-3} \Rightarrow R_1 = 7.56\ mm$ //

3. $\because m_\ell = -\ell, -\ell+1, \cdots, \ell-1, \ell \quad \therefore m_\ell = -2, -1, 0, 1, 2$ //

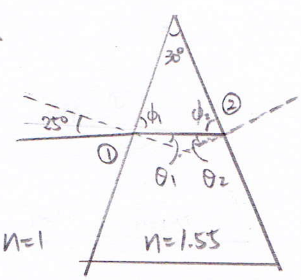
4. $R = R_0 e^{-\lambda t}$, $T_{1/2} = \frac{0.693}{\lambda} = 6.05\,hr \Rightarrow \lambda = 0.115\ hr^{-1} \quad \therefore R(t=2) = 1.1\times10^4 \times e^{-0.115\cdot 2} = 8.7\times10^3\ Bq$ //

5. $2t = (m+\frac{1}{2})\lambda_n = (m+\frac{1}{2})\frac{\lambda}{n}$ (longest $\lambda \Rightarrow m=0$) $\therefore 2t = \frac{1}{2}\frac{\lambda}{n} \Rightarrow \lambda = 4nt = 4\times1.3\times125 = 650\,nm$ //

6. $f_{beat} = |f - 264| = 2 \quad \therefore f = 262$ or $266\ Hz$ //

7. Refer to the lecture note / textbook.

8.



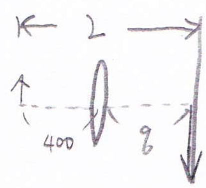
interface ① : $1\cdot\sin25° = 1.55\sin\theta_1 \Rightarrow \theta_1 = 15.8°$

$\Rightarrow \phi_1 = 90° - \theta_1 = 74.2° \Rightarrow \phi_2 = 180° - 30° - \phi_1 = 75.8°$

$\Rightarrow \theta_2 = 90° - \phi_2 = 14.2°$

interface ② : $1.55\cdot\sin14.2° = 1\cdot\sin\theta_3 \Rightarrow \theta_3 = 22.3°$ //

9.



$p = 400$, $M = -\frac{q}{p} = -3 \Rightarrow q = 3p = 1200\ mm \Rightarrow L = q+p = 1600\ mm$

and $\frac{1}{400} + \frac{1}{1200} = \frac{1}{f} \Rightarrow f = 300\ mm$

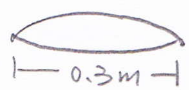
To make $M = -5 \Rightarrow -\frac{q}{p} = -5 \Rightarrow q = 5p$

$\because q+p = 1600 \Rightarrow 5p+p = 6p = 1600 \Rightarrow p = 266\,mm$ //

10. $f = 1.25,\ \lambda = 1 \Rightarrow v = f \cdot \lambda = 1.25\ m/s$ is constant $/\!/$

$\Rightarrow$ When $f = 2.5 \Rightarrow \lambda = \frac{v}{f} = \frac{1.25}{2.5} = 0.5\ (m)$ $/\!/$

11. ① Find the speed of the wave along the string:



$L = 0.3 = \frac{\lambda}{2} \Rightarrow \lambda = 0.6m \Rightarrow v = f \cdot \lambda = 118.2\ m/s$

② Find the length of the string when the finger is at the position to play A (440Hz):

$\lambda_A = \frac{v}{f_A} = \frac{118.2}{440} = 0.269\,m$, $L_A = \frac{\lambda}{2} \Rightarrow L_A = 0.134\ m = 13.4\ cm$

③ Moving the finger by ± 0.5cm changes L from $13.4 - 0.5 = \underline{12.9\,cm}$ to $13.4 + 0.5 = \underline{13.9\ cm}$

(i) When $L = 12.9\ cm = 0.129\,m = \frac{\lambda}{2} \Rightarrow \lambda = 0.258\,m \Rightarrow f = \frac{118.2}{0.258} = 458\ Hz$

(ii) When $L = 13.9\ cm = 0.139\,m = \frac{\lambda}{2} \Rightarrow \lambda = 0.278\ m \Rightarrow f = \frac{118.2}{0.278} = 425\ Hz$

$\therefore$ 425–458 Hz $/\!/$

12. $\frac{1}{3600}$ degree $= 4.85 \times 10^{-6}$ rad. $\theta_{min} = 1.22 \cdot \frac{\lambda}{d} \Rightarrow 4.85 \times 10^{-6} = 1.22 \cdot \frac{550 \times 10^{-9}}{d}$

$\Rightarrow d = 0.138\ m = 13.8\ cm$ $/\!/$

13. $\lambda_{\text{de Broglie}} = \frac{h}{p} = \frac{h}{\sqrt{2mE}} \Rightarrow \lambda^2 = \frac{h^2}{2mE} \Rightarrow E = \frac{h^2}{2m\lambda^2} = \frac{(6.626 \times 10^{-34})^2}{2 \times 9.11 \times 10^{-31} \times (0.85 \times 10^{-10})^2}$

$= 3.34 \times 10^{-17}\ J = 208\ eV$ $/\!/$

14. ${}^{240}_{94}Pu \longrightarrow {}^{236}_{92}U + {}^{4}_{2}\square$ $\therefore {}^{4}_{2}He$ $/\!/$

15. $E_0 = 75\,keV = 1.2 \times 10^{-14}\,J = \frac{hc}{\lambda_0} \Rightarrow \lambda_0 = 1.66 \times 10^{-11}\,(m)$

$\Delta\lambda = \lambda' - \lambda_0 = \frac{h}{m_e c}(1-\cos\theta) = \frac{6.626 \times 10^{-34}}{9.11 \times 10^{-31} \times 3 \times 10^8} \times (1 - \cos 75°) = 1.80 \times 10^{-12}\,(m)$

$\therefore \lambda' = \Delta\lambda + \lambda_0 = 1.84 \times 10^{-11}\,(m)$ //

16. ${}^{197}_{79}Au$ has 79 protons (1.007276u) and 118 neutrons (1.008665 u)

$\therefore E_b = (1.007276 \times 79 + 1.008665 \times 118 - 196.966543) \times 931.494$

$= (79.574804 + 119.02247 - 196.966543) \times 931.494$

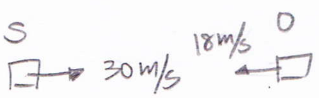
$= 1519\ MeV$ $\therefore$ Binding energy per nucleon $= \frac{1519}{197} = 7.71\ MeV$ //

17. $T = 2\pi\sqrt{\frac{L}{g}}$

18. $m = 0.2\,kg,\ k = 40\,N/m,\ A = 0.04\,m \Rightarrow E = \frac{1}{2}kA^2 = \frac{1}{2} \times 40 \times 0.04^2 = 0.032\,J$

At the equilibrium point, $U_k = 0$ $\therefore E = K = \frac{1}{2}mv^2$ $\therefore v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \cdot 0.032}{0.2}} = 0.57\,m/s$ //

19.



$f' = \frac{340+30}{340-18} \times 262 = 301\ Hz$ //

20. $T_{1/2} = \frac{0.693}{\lambda} = 2\ min \Rightarrow \lambda = 0.3465\ min^{-1}$

$N = N_0 e^{-\lambda t} \Rightarrow \frac{N}{N_0} = e^{-\lambda t}$ $\therefore \frac{N}{N_0} = \frac{1}{1000} = 0.001 = e^{-0.3465 \cdot t}$

$\ln(0.001) = \ln(e^{-0.3465 \cdot t}) = -0.3465t$ $(\ln 0.001 = -0.6908)$

$\Rightarrow t = 20$ mins //