Physics 1B
Electricity & Magnetism

Frank Wuerthwein (Prof)

Edward Ronan (TA)

UCSD

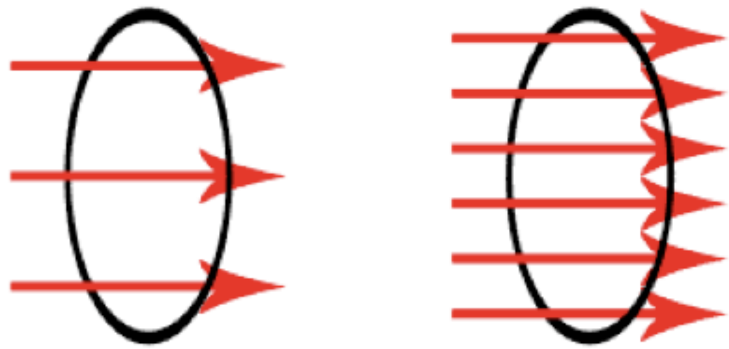
Outline of Today

- Electric Flux & Gauss Law
- Quiz preparation:
  - A reminder of geometry
- Quiz 1 on Friday January 27th will cover Chapter 19 only.

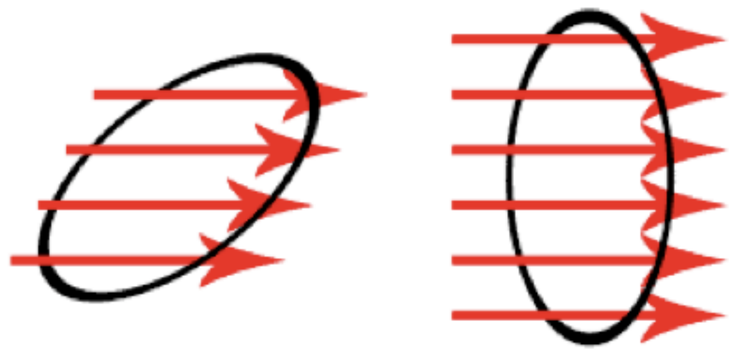
Electric Flux

- Flux is the amount of something that flows through a given area.
- Electric flux, Φ_E, measures the amount of electric field lines that passes through a given area.
- The electric flux depends on:
  - How big the area is.
  - How strong the electric field is.
  - The angle between the area and the electric field.

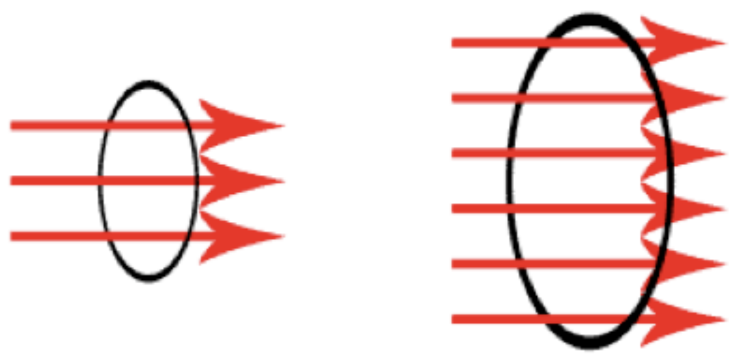
Electric Flux



Flux is proportional to the density of flow.



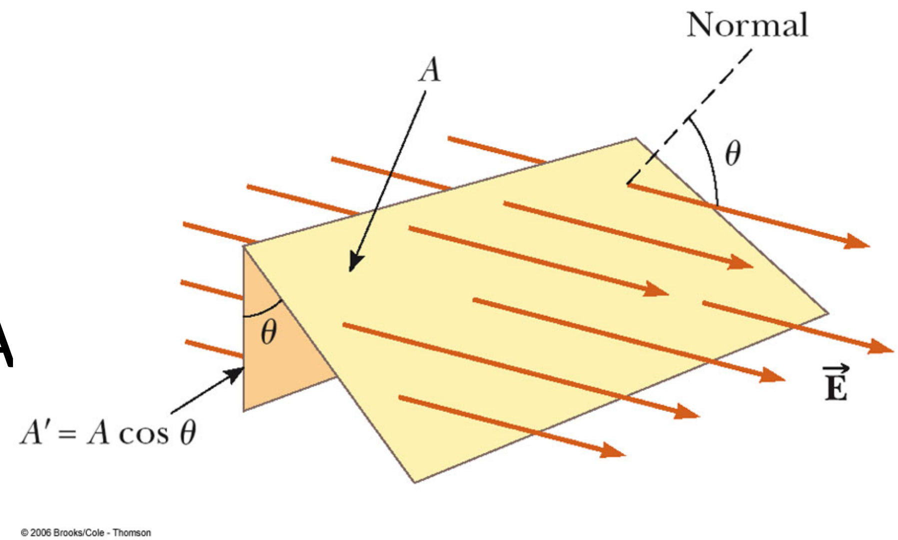
Flux varies by how the boundary faces the direction of flow



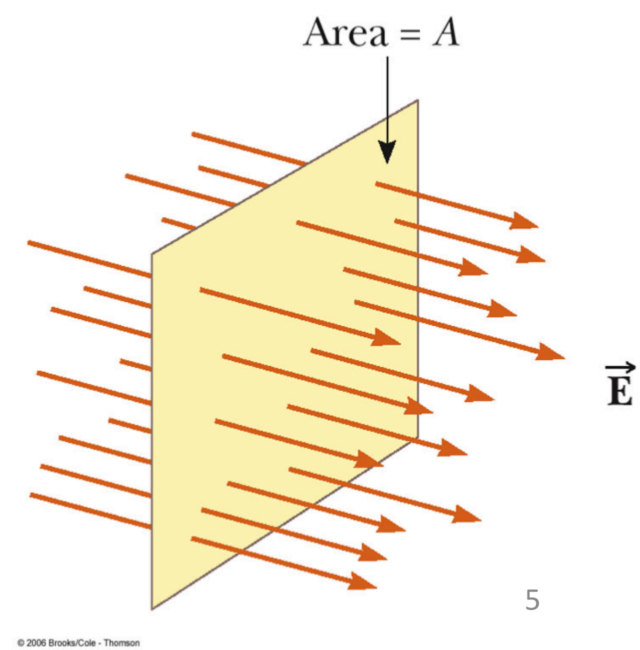
Flux is proportional to the area within the boundary.

Electric Flux

- In order to better understand the angle between E and A, we define the direction of A by a normal to A.



Electric flux will depend on E, A, and θ (where θ is the angle between E and the normal to A).



$$\Phi_E = \vec{E}\vec{A} = EA\cos\theta$$

Definition of a closed surface

- A closed surface is one that encloses a small volume and has no openings (it distinctly divides a space into inside and outside regions).
- Examples of a closed surface are spheres, boxes, cylinders, etc.

Flux in and out of a closed surface

- flux lines passing into the interior of the volume are negative.
  - Imagine the volume to contain a sink into which the flow vanishes. Thus negative.
- Flux lines passing out of the interior of the volume are positive.
  - Imagine the volume to contain a source from which flow appears. Thus positive.

Gauss' Law

- Gauss' Law states that the electric flux through any closed surface is equal to the net charge, Q_{inside}, inside the surface divided by the constant ε_o.

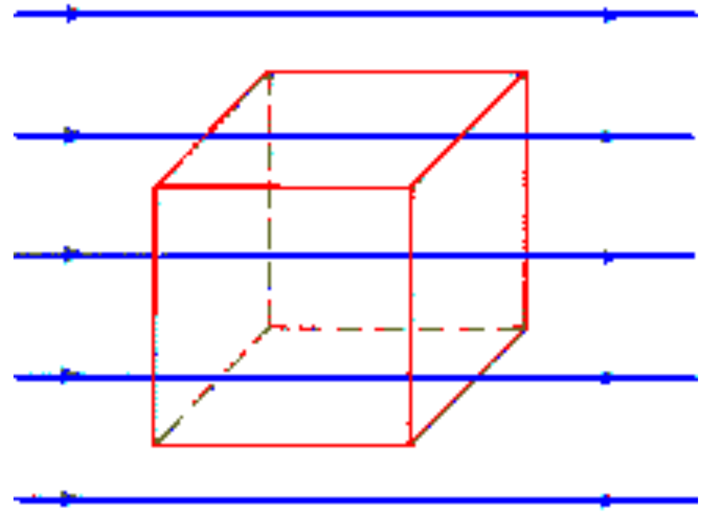
$$\Phi_E = \frac{Q_{inside}}{\varepsilon_o}$$

where $\varepsilon_o = 8.85 \times 10^{-12} C^2/(Nm^2)$ and is called the permittivity of free space.

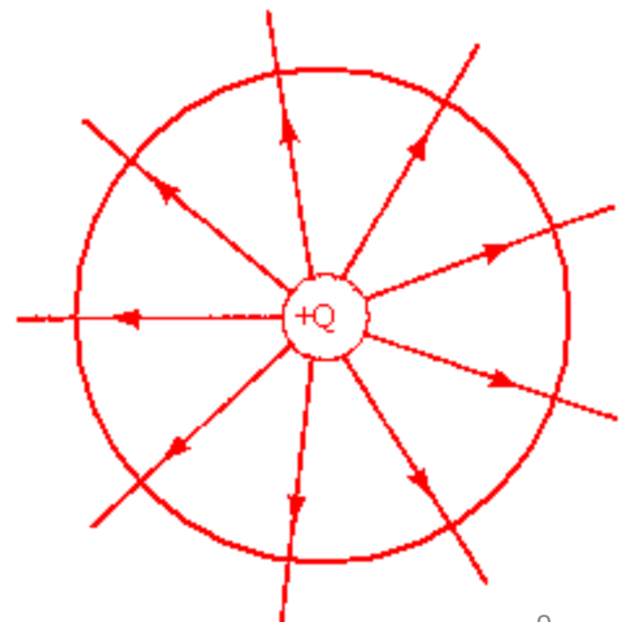
Basically, Gauss' Law tells us that the only way you get more lines coming out than going into a closed surface, is to have a positive charge inside.

Gauss' Law

- What if there were no charges present inside the closed Gaussian surface, what would the electric flux be?



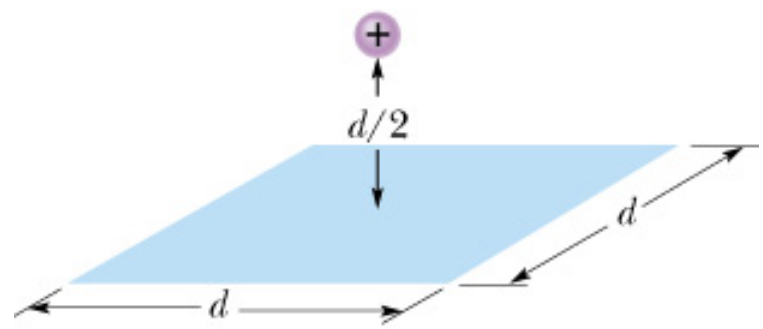
The amount of field lines that enter the surface equal the amount of field lines that leave the surface. This leads to a net electric flux of zero.



But if a net positive charge is present, then there is a source of electric field lines and the electric flux cannot be zero.

Gauss' Law

- <u>Example</u>
- In the figure below, a proton is a distance d/2 directly above the center of a square of side d. Assume d = 2.0cm. What is the magnitude of electric flux through the square?



<u>Answer</u>

Define a coordinate system.

Choose the proton as r = 0. Any outward distance would be positive.

Gauss' Law

<u>Answer</u>

Our first thought is to use the equation for electric flux:

$$\Phi_E = EA\cos\theta$$

Will the strength of the electric field be the same in magnitude at all points on the square face?

No, r will be different between the middle of the square and the sides of the square.

So we cannot use this equation.

As with nearly all Gauss' Law problems we will need to exploit symmetry to solve it.

Gauss' Law

<u>Answer</u>

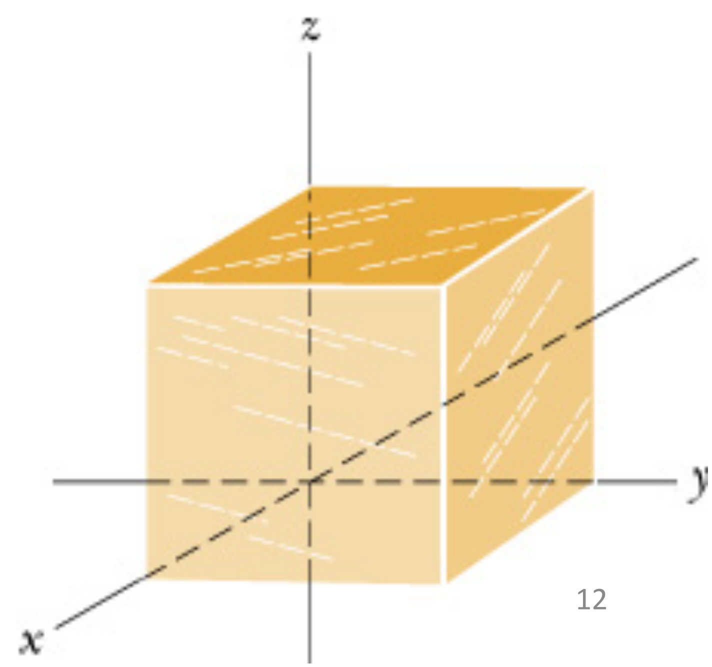
If we are going to use Gauss' Law we must choose a Gaussian surface.

Since this problem has no spherical symmetry we must try to choose another Gaussian surface.

What type of surface seems appropriate?

A square centered on the proton seems the most appropriate.

The square should have sides of distance d with the origin at its' center.



Gauss' Law

Answer

Next, apply Gauss' Law.

How much charge is located inside the Gaussian sphere?

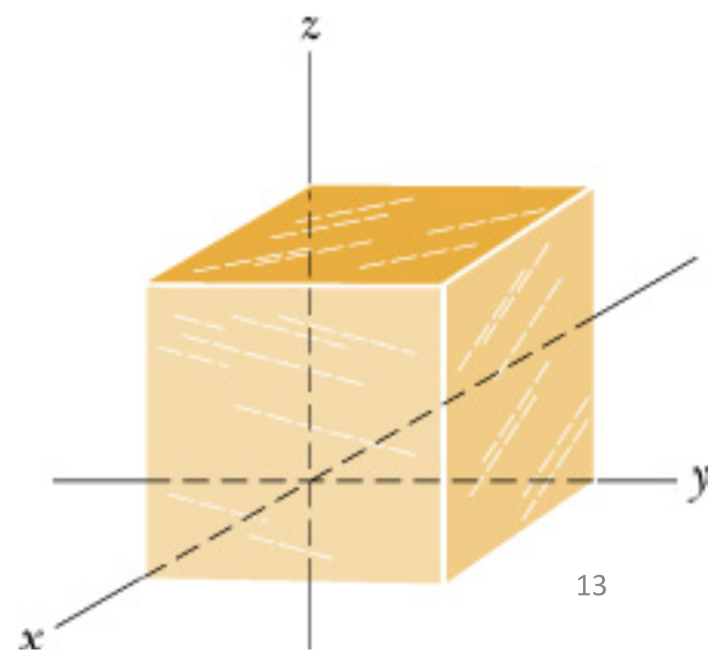
+e.

$$\Phi_E = \frac{Q_{inside}}{\varepsilon_o}$$

That is the total flux going through the entire cube.

But how much is going through just the bottom face?

1/6 of that (since the cube has six symmetric faces).



Gauss' Law

Answer

So, for one face we have:

$$\Phi_E = \left(\frac{1}{6}\right)\frac{+e}{\varepsilon_o}$$

$$\Phi_E = \left(\frac{1}{6}\right)\frac{\left(1.602\times 10^{-19}\,\mathrm{C}\right)}{8.85\times 10^{-12}\,\mathrm{C^2}/\mathrm{N\cdot m^2}} = 3.02\times 10^{-9}\,\mathrm{N\cdot m^2}/\mathrm{C}$$

Did it matter what value d was?

No, not at all.

Applications of Gauss Law

- Calculate the E field of a point charge at a point a distance r away from the charge.
  - Chapter 19 example 19.9
- Calculate the E field of an infinite straight line with uniform charge density λ
  - Chapter 19 example 19.11

E field of a point charge

$$Flux = \vec{E}\vec{A} = EA = \frac{q}{\varepsilon_0}$$

$$A = 4\pi r^2$$

$$\Rightarrow E = \frac{q}{4\pi r^2 \varepsilon_0}$$

E field of a straight line with uniform charge density

$$Flux = \vec{E}\vec{A} = EA = \frac{\lambda l}{\varepsilon_0}$$

$$A = 2\pi r l$$

$$\Rightarrow E = \frac{\lambda}{2\pi r \varepsilon_0} = 2k_e \frac{\lambda}{r}$$

Why do the disks at the end of the cylinder not contribute to the flux?

Reminder of Geometry (I)

- The angular bisectors of all three angles in an equilateral triangle meet in one point.
- This point is 2/3 the length of the bisector away from each of the three corners.
- What's the force on a test charge in this point if there is the same charge +Q at each of the three corners?
- Recall: a tetrahedron is a 3 dimensional object for which the surface is composed of 4 equilateral triangles.

Reminder of Geometry (II)

- The diagonals of a square intersect in a point.
- This point is the same distance away from each corner.
- Is the same still true for a rectangle?

Reminder of Geometry (III)

- All points on a circle are the same distance away from the circles center.
- All points on a sphere are the same distance away from the center of the sphere.

- I suggest you refresh your memory about the equations for the surface of cube, sphere, tetrahedron.

For Next Time (FNT)

- Start reading chapter 20
- Finish working on the homework for chapter 19.