Physics 1B
Electricity & Magnetism

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Outline of today

- Chapter 21
    - Kirchhoff’s rules
    - RC circuits

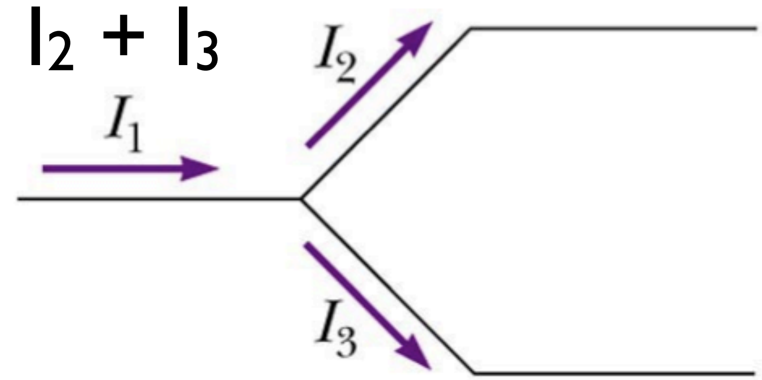
Kirchhoff's Rules

- Kirchhoff's Loop Rule
- The sum of potential differences across all the elements around any closed circuit must be zero.

$$\sum \Delta V_{loop} = \Delta V_1 + \Delta V_2 + \Delta V_3 \ldots = 0$$

- Kirchhoff's Junction Rule
- The sum of the currents entering any junction must equal the sum of the currents leaving.

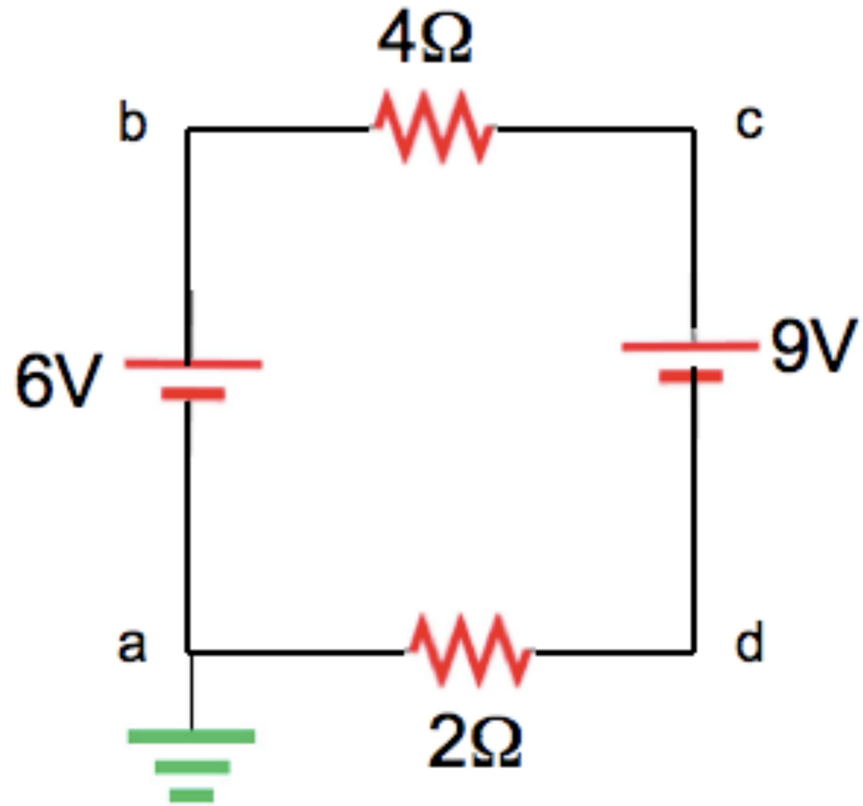
$I_1 = I_2 + I_3$



(a)

Circuits

- Example
- For the following circuit diagram, what is the value of the potential at points a, b, c, and d?



Answer

First, we need to identify which variables we need.

Since we want potential at the four points, we will want the current (?) that passes through the entire circuit to identify the different potential drops.

Answer

Circuits

Let's choose the direction of the current as counterclockwise.
Let's choose voltage loop directions as counterclockwise as well:

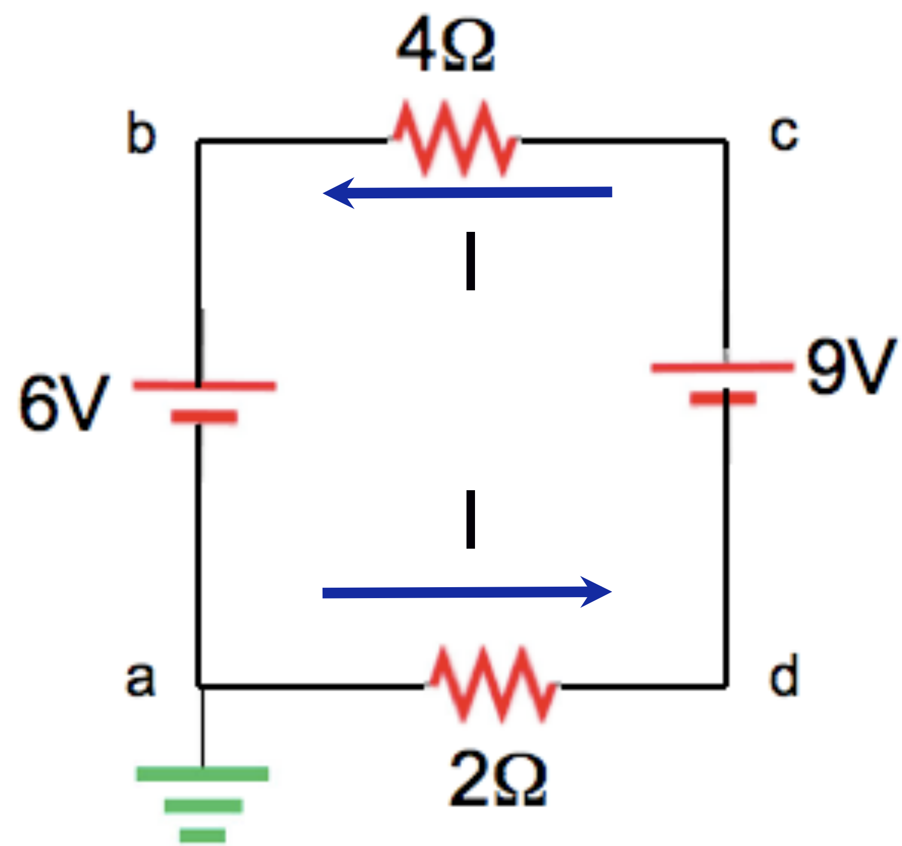
Now, by applying the loop rule on the loop we get:

$$\sum V_{loop} = \Delta V_{2\Omega} + \Delta V_{9V} + \Delta V_{4\Omega} + \Delta V_{6V} = 0$$

$$-I(2\Omega) + 9\text{V} - I(4\Omega) - 6\text{V} = 0$$

$$3\text{V} - I(6\Omega) = 0$$

$$I = \frac{3\text{V}}{6\Omega} = 0.5\text{A}$$



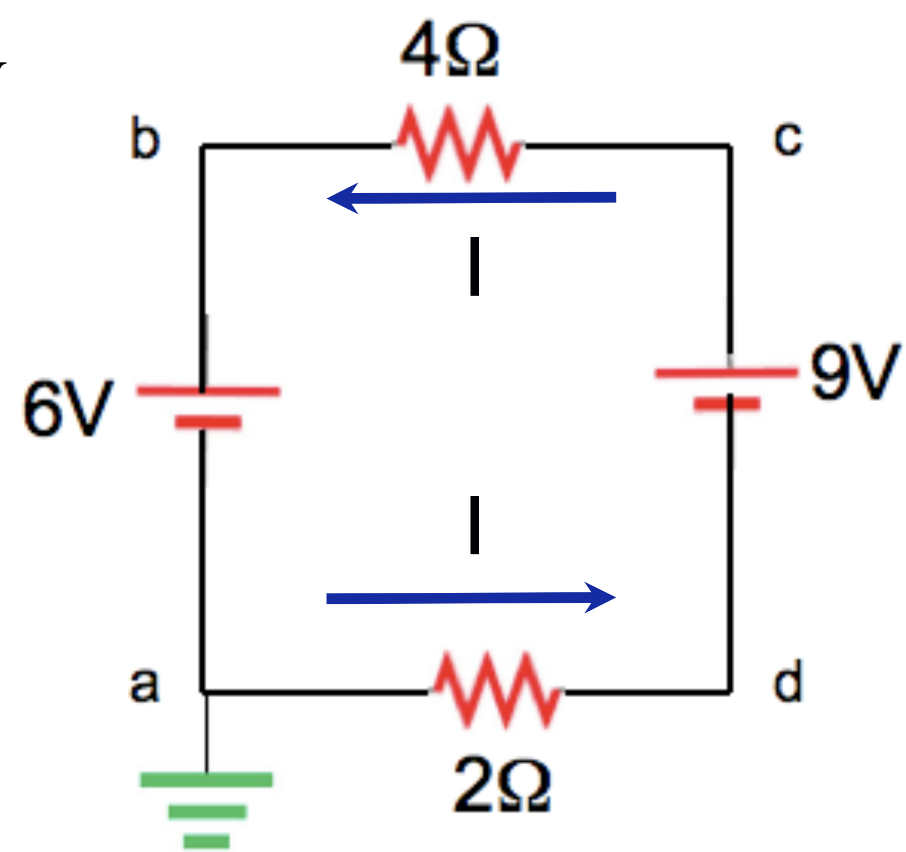
Answer

Circuits

By the definition of the "ground," the potential at point a must be zero ($V_a = 0$).

Now, moving counterclockwise from point a, we find that we get a drop in voltage by moving through the 2Ω resistor:

$$V_d = 0\text{V} - IR = 0\text{V} - 0.5A(2\Omega) = -1\text{V}$$



Next, we move through the 9V battery to point c:

$$V_c = -1\text{V} + 9\text{V} = 8\text{V}$$

Then, moving through the 4 Ω resistor to point b:

$$V_b = 8\text{V} - IR = 8\text{V} - 0.5A(4\Omega) = 6\text{V}$$

<u>Answer</u>

Circuits

Finally, moving through the 6V battery and going back to point a:

$$V_a = -6\text{V} + 6\text{V} = 0\text{V}$$

So, the answers become:

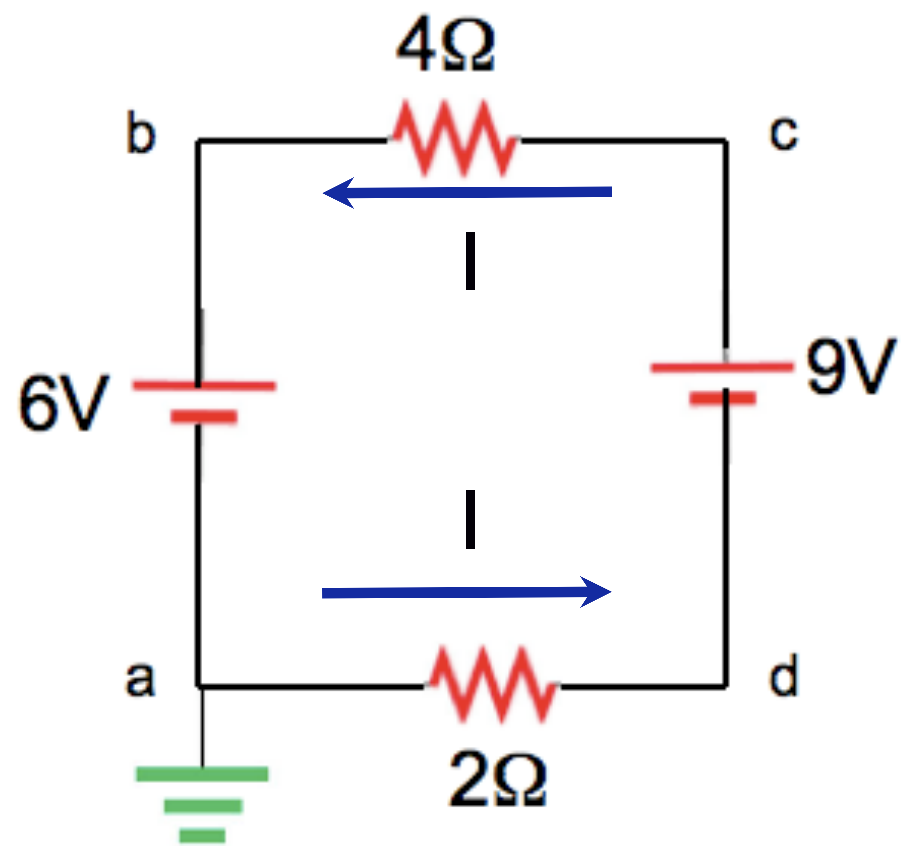
$V_a = 0V$

$V_b = 6V$

$V_c = 8V$

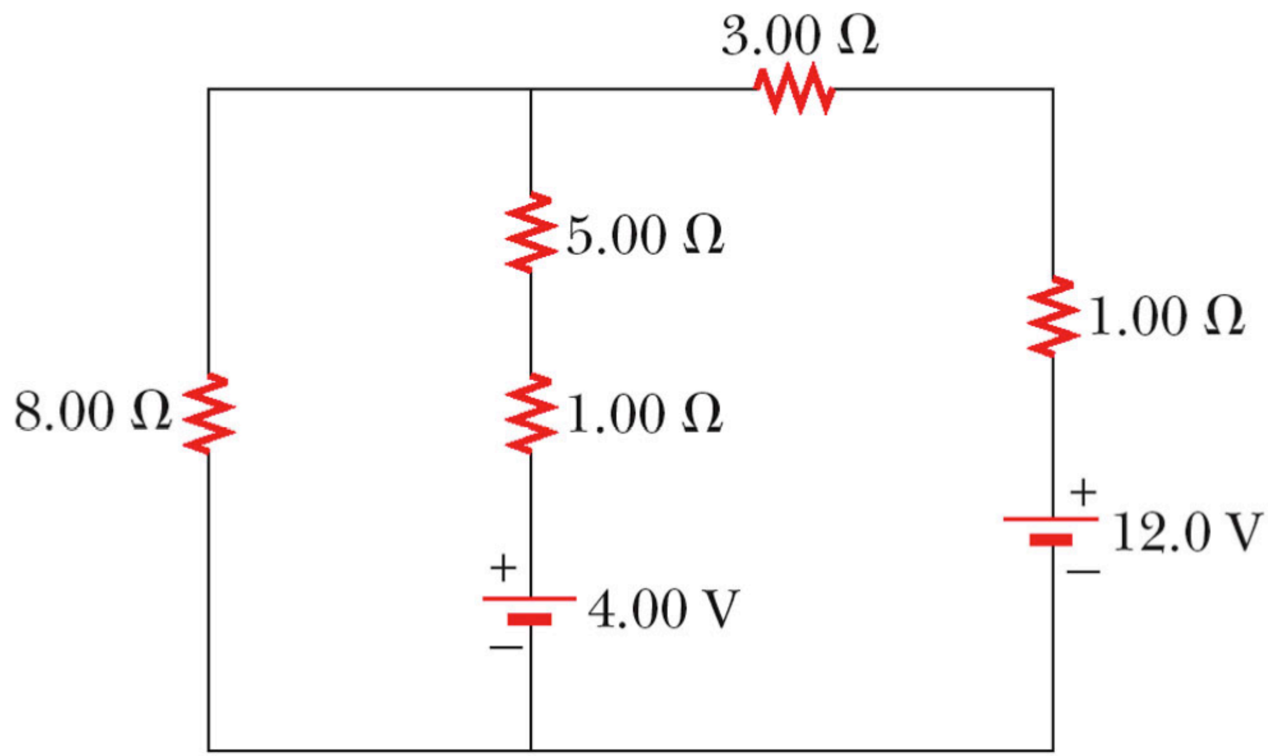
$V_d = -1V$

It is possible to have negative voltage values (even with a ground), you are just establishing a coordinate system.



Circuits

- <u>Example</u>
- For the following circuit diagram, what is the power dissipated by the 8Ω resistor?



<u>Answer</u>

First, we need to identify which variables we need.

Since we want power dissipated by the 8Ω resistor, we will want the resistance (8Ω) and the current (?) that passes through it.

Circuits

Answer

Let's reduce what we can by equivalent resistance:

We can combine the two pairs of resistors in series.

$$R_{eq,left} = 5\Omega + 1\Omega = 6\Omega$$

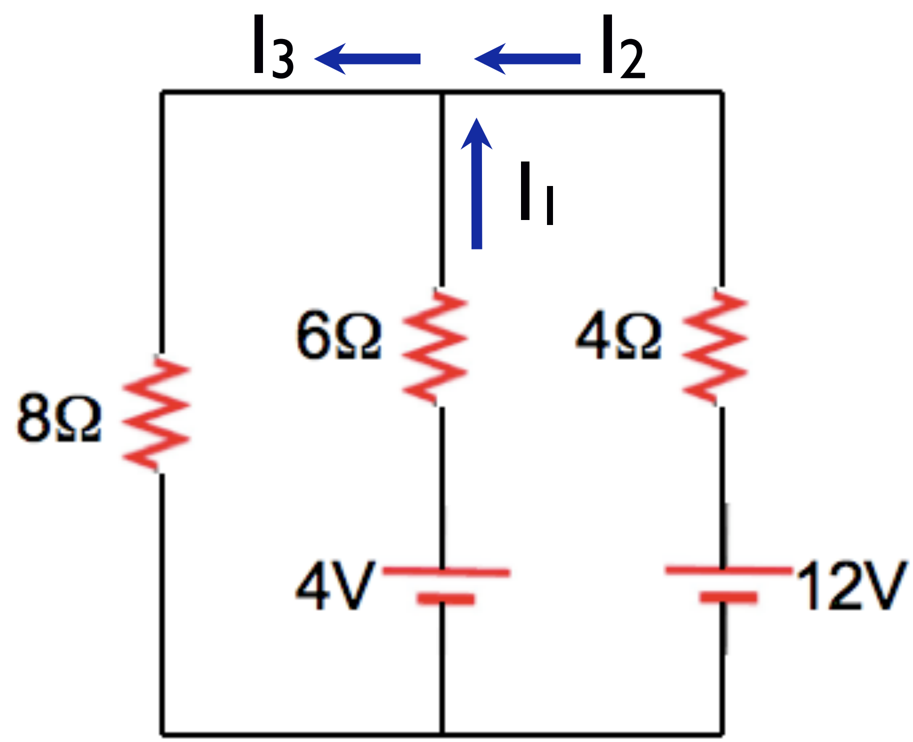
$$R_{eq,right} = 1\Omega + 3\Omega = 4\Omega$$

Then, we can identify the directions for currents at the top junction:

Choose up for I_1.

To the left for I_2.

To the left for I_3. <-want



Now, by applying the junction rule we see that:

$$I_3 = I_1 + I_2$$

Circuits

Answer

Let's choose voltage loop directions (need I_1 and I_2):

We can make an inner left loop moving counterclockwise and an out outer loop moving counterclockwise.

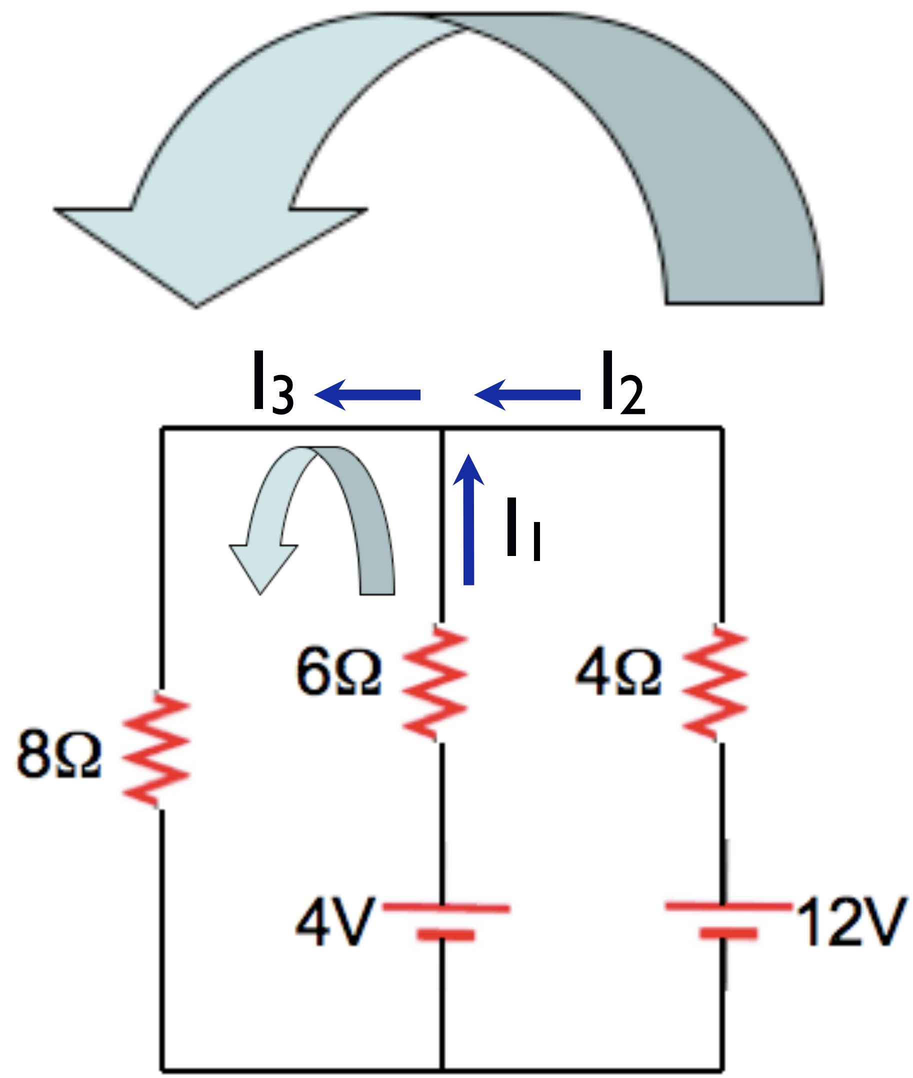
Now, by applying the loop rule on inner loop we get:

$$\sum V_{loop} = \Delta V_{4V} + \Delta V_{6\Omega} + \Delta V_{8\Omega} = 0$$

$$4\text{V} - I_1(6\Omega) - I_3(8\Omega) = 0$$

$$I_1(6\Omega) = 4\text{V} - I_3(8\Omega)$$

$$I_1 = \tfrac{2}{3}\text{A} - I_3\left(\tfrac{4}{3}\right)$$



Circuits

<u>Answer</u>

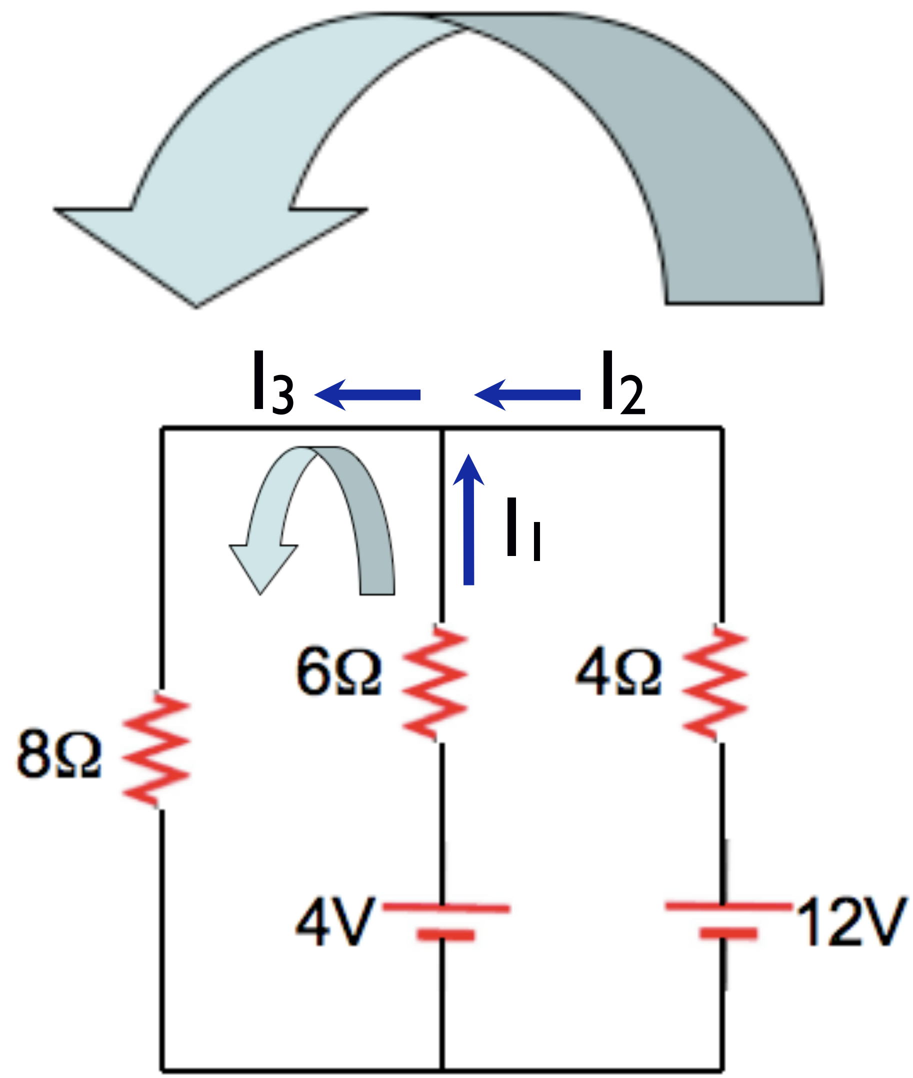
Now, by applying the loop rule on outer loop we get:

$$\sum V_{loop} = \Delta V_{12V} + \Delta V_{4\Omega} + \Delta V_{8\Omega} = 0$$

$$12V - I_2(4\Omega) - I_3(8\Omega) = 0$$

$$I_2(4\Omega) = 12V - I_3(8\Omega)$$

$$I_2 = 3A - I_3(2)$$



Next, we can combine these two loop equation results with the junction equation result to give us our I_3 value.

Answer

Circuits

Start with:

$$I_3 = I_1 + I_2$$

$$I_3 = \left(\tfrac{2}{3}\text{A} - I_3\left(\tfrac{4}{3}\right)\right) + \left(3\text{A} - I_3(2)\right)$$

$$I_3 = \left(\tfrac{2}{3}\text{A} + 3\text{A}\right) + \left(-I_3\left(\tfrac{4}{3}\right) - I_3(2)\right)$$

$$I_3 = \left(\tfrac{11}{3}\text{A}\right) - I_3\left(\tfrac{10}{3}\right)$$

$$I_3\left(\tfrac{13}{3}\right) = \tfrac{11}{3}\text{A}$$

$$I_3 = \left(\tfrac{3}{13}\right)\tfrac{11}{3}\text{A} = \tfrac{11}{13}\text{A} = 0.846\text{A}$$

Answer

Circuits

Now, to solve for the power dissipated by the 8 Ω resistor turn to:

$$P_{dis} = I^2 R$$

$$P_{dis} = I_3^2 R_8 = (0.846\text{A})^2 (8\Omega) = 5.73\text{W}$$

But, what if we chose the wrong direction of I_1, I_2, or I_3 when we first started the problem?

Would our final answer be wrong?

Let's examine the value of I_1 in this example to check if we got it right.

Circuits

From before:

$$I_1 = \frac{2}{3}\text{A} - I_3\left(\frac{4}{3}\right)$$

$$I_1 = \frac{2}{3}\text{A} - \left(\frac{11}{13}\text{A}\right)\left(\frac{4}{3}\right)$$

$$I_1 = 0.667\text{A} - 1.128\text{A} = -0.461\text{A}$$

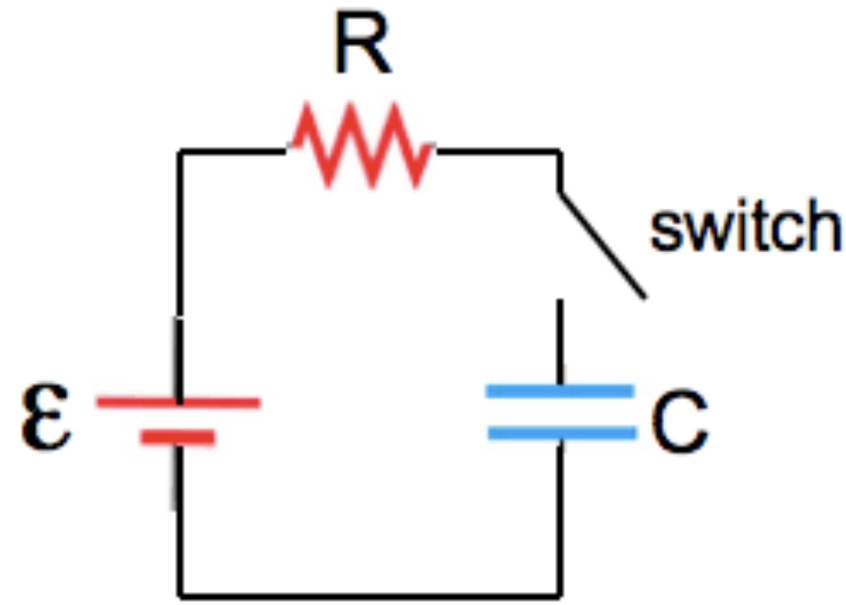
I_1 is negative, so I_1 actually went down, not up.

But that didn't affect our final answer.

Basically, when we chose our current directions, we chose a coordinate system, I_1 happened to go in the negative direction.

RC Circuits

- Up until now, we have only had capacitors or resistors as our circuit elements, but what would happen if we had both?
- The simplest such circuit is:



What will happen when the switch is closed?

A current will start to flow through the resistor and the capacitor will begin to charge.

Charges from one plate of the capacitor will flow to the other.

RC Circuits

- Will the current be constant?
- No, as more and more charge build up, the current will slow down until it becomes zero.
- This means that the current, I, is time dependent.
- The charge, Q, on the capacitor plates will also be time dependent.
- As time progresses, the charge will increase until it reaches some final value, Q_f.

RC Circuits

- Perform a Kirchoff loop over the circuit to find Q_f.

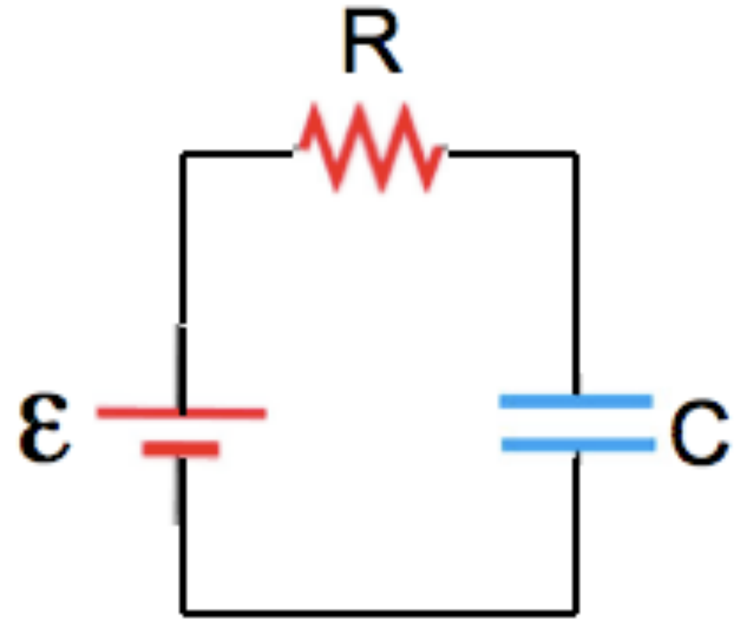
$$\sum \Delta V_{loop} = \Delta V_1 + \Delta V_2 + \Delta V_3 = 0$$

$$\mathcal{E} - \Delta V_R - \Delta V_C = 0$$

$$\mathcal{E} - IR - \frac{Q}{C} = 0$$

When the capacitor is fully charged, the current, I, will become zero.

$$Q_f = C\mathcal{E}$$



RC Circuits

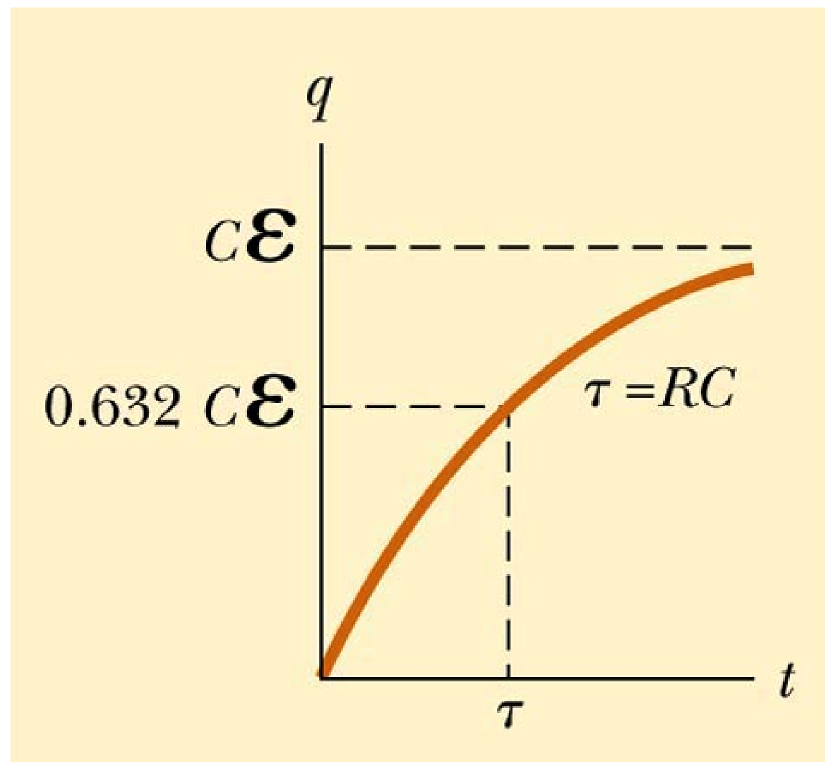
- But if we would like to find the charge on the capacitor plate at any time, t, we have to perform some math.

$$\mathcal{E} - IR - \frac{Q}{C} = 0 \qquad \mathcal{E} - \frac{\Delta Q}{\Delta t}R - \frac{Q}{C} = 0$$

This is a differential equation, the solution is (to find Q for a charging capacitor):

$$Q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

The capacitor charges exponentially as a function of time:



Concept Question

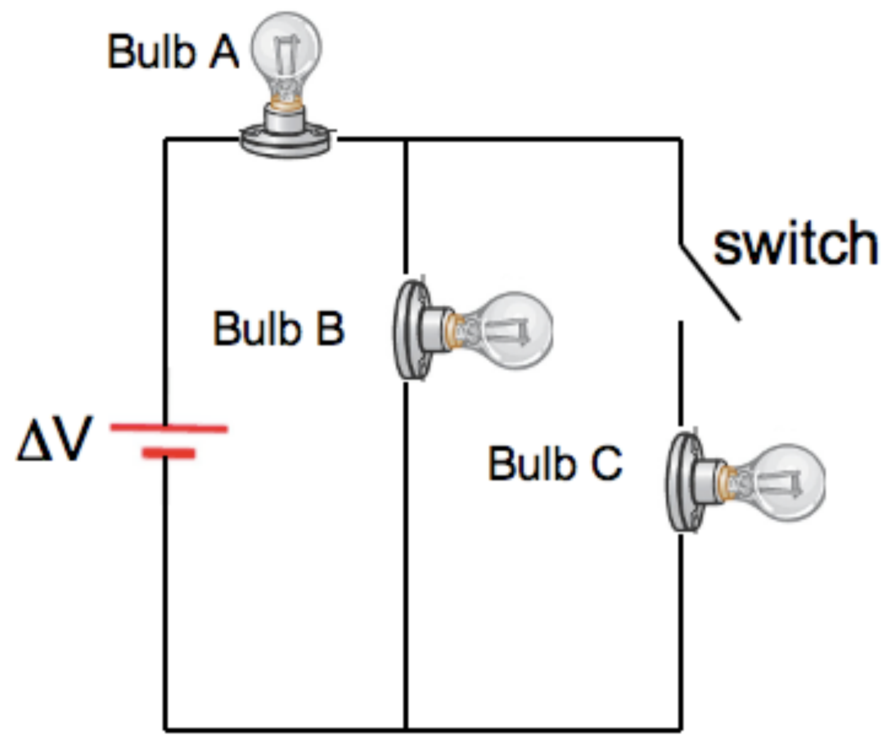
- Initially light bulbs A and B are hooked up in series and connected to a voltage source. If the switch is closed, light bulb C will become hooked up in parallel with light bulb B. What happens to the brightness of light bulbs A and B when the switch is closed? All light bulbs have the same resistance in this circuit.

A) Bulb A remains the same and Bulb B gets dimmer.

B) Bulb A remains the same and Bulb B remains the same.

- C) Bulb A gets brighter and Bulb B gets dimmer.

- D) Bulb A gets dimmer and Bulb B gets dimmer.



Household Circuits

- In your house, you have parallel wiring between your appliances.
- If you had wiring in series, if one light bulb went out the whole house would stop consuming power.
- Each appliance is independent of the others.
- The power company supplies 120V of electric potential.
- Also, if current were to get too high (~100A), the wires would physically melt.

Household Circuits

- To avoid this, fuses are available on every circuit.
- Fuses disconnect circuits (make them open circuits) when current gets too high (Fuses typically blow at about 10A).
- Caution: Never become part of a household circuit. Death can occur at about 1A.
- Especially be careful of cuts on the skin around electricity.
- Dry skin has a resistance of about 1MΩ, but just under your skin you have a resistance of about 10Ω.

For Next Time (FNT)

- Continue homework for Chapter 21
- Finish reading Chapter 21