Track Parameters

Motion of charged particle in a <u>Constant</u> Magnetic Field

Lorentz Force Law: $\vec{F} = \dfrac{d\vec{p}}{dt} = \dfrac{q}{c}\, \vec{v} \times \vec{B}$

Since

$$W = \int \vec{F} \cdot d\vec{\ell}$$

$$= \frac{q}{c} \int (\vec{v} \times \vec{B}) \cdot d\vec{\ell}$$

$$= -\frac{q}{c} \int \vec{B} \cdot (\vec{v} \times d\vec{\ell}) \qquad \text{but } d\vec{\ell} \parallel \vec{v}$$

$$= 0 \qquad (\text{Magnetic Field does no work})$$

$\Rightarrow$

$$|\vec{p}| = p = \text{const}$$

$$\gamma = \text{const} \qquad (\text{Lorentz factor})$$

$$v = \frac{d|\vec{x}|}{dt} \qquad (\text{speed})$$

Let's parameterize w/respect to s = arc length

$$ds = v\, dt$$

we know,

$$\vec{v} = \frac{d\vec{x}}{dt} = v \underbrace{\frac{d\vec{x}}{ds}}_{\hat{\alpha}}$$

$$= v\hat{\alpha} \quad , \quad \hat{\alpha} \equiv \frac{d\vec{x}}{ds}$$

so $\vec{p}$ becomes,

$$\vec{p} = \gamma m \vec{v} = \gamma m v \hat{\alpha}$$

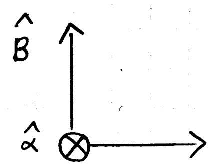
$$\frac{d\vec{p}}{dt} = v\frac{d\vec{p}}{ds} = \frac{q}{c}\, \vec{v} \times \vec{B}$$

$$v p \frac{d\hat{\alpha}}{ds} = \frac{q}{c}\, v\, (\hat{v} \times \vec{B})$$

$$\frac{d\hat{\alpha}}{ds} = \frac{1}{c}\left(\frac{qB}{p}\right)(\hat{\alpha} \times \hat{B})$$

let $\rho = \frac{pc}{qB}$ $\qquad \frac{[E]}{[F]} = \frac{[F \cdot L]}{[F]} = [L]$

$\uparrow$ instananeous radius of curvature.



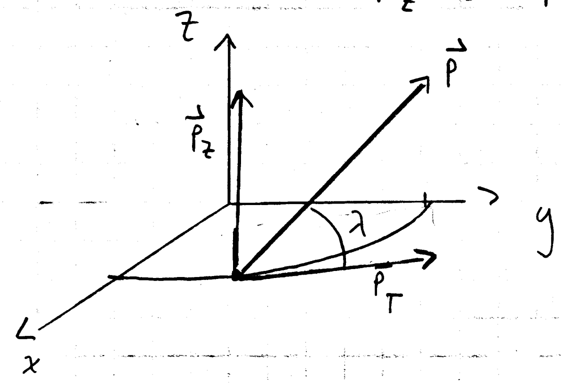
$\frac{d\vec{\alpha}}{ds} \propto \hat{\alpha} \times \hat{B} \propto \hat{\rho}$

=> The momentum vector describes a circle of radius ρ about $\hat{B}$.

let $\vec{B} = B\hat{z}$

=> $F_z = 0$ => $p_z = \text{const}$

$p_z = \vec{p} \cdot \hat{z} = p \sin\lambda$



$\lambda = \frac{p_z}{p_T} = \frac{\pi}{2} - \theta$

$\uparrow$ polar angle

$$z - z_0 = \underbrace{\alpha_{z0}\, s}_{\text{projection of } s \text{ along } z\text{-axis}}$$

$\hat{\alpha} = \alpha_x \hat{x} + \alpha_y \hat{y} + \alpha_z \hat{z}$

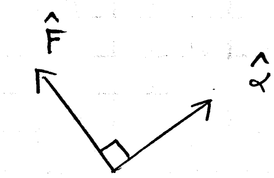
with $\alpha_x^2 + \alpha_y^2 + \alpha_z^2 = 1$

So far we have two track parameters

$\frac{q}{p}$, λ

$\nwarrow$ don't use curvature since $\vec{\nabla} \cdot \vec{B} \neq 0$ for real detector

Now consider $x-y$ plane:



$$\frac{d}{ds}\begin{pmatrix}\alpha_x \\ \alpha_y\end{pmatrix} = \frac{1}{\rho}\begin{pmatrix}\alpha_y \\ -\alpha_x\end{pmatrix}$$

differentiate again (to decouple)

$$\frac{d^2}{ds^2}\alpha_x = \frac{1}{\rho}\frac{d\alpha_y}{ds} = -\frac{1}{\rho^2}\alpha_x$$

$$\frac{d^2}{ds^2}\alpha_y = -\frac{1}{\rho^2}\alpha_y$$

=> Solution

$$\alpha_x = A\cos\left(\frac{s}{\rho}\right) + B\sin\left(\frac{s}{\rho}\right)$$

$$\alpha_y = C\cos\left(\frac{s}{\rho}\right) + D\sin\left(\frac{s}{\rho}\right)$$

let $\phi \equiv \phi(s) = \frac{s}{\rho}$; set $\hat{\alpha}_0 = \begin{pmatrix}\alpha_{x0} \\ \alpha_{y0}\end{pmatrix}$

initial momentum => $A = \alpha_{x0}$

$C = \alpha_{y0}$

$$\frac{d}{ds}\hat{\alpha}_0 = \frac{1}{\rho}\begin{pmatrix}\alpha_{y0} \\ -\alpha_{x0}\end{pmatrix}$$

$$\frac{d}{ds}\alpha_x = -\frac{A}{\rho}\sin\left(\frac{s}{\rho}\right) + \frac{B}{\rho}\cos\left(\frac{s}{\rho}\right)$$

$$\frac{d}{dy}a_y = -\frac{C}{\rho}\sin\left(\frac{s}{\rho}\right) + \frac{D}{\rho}\cos\left(\frac{s}{\rho}\right)$$

initial acceleration => $\frac{B}{\rho} = \frac{\alpha_{y0}}{\rho}$

$$\frac{C}{\rho} = -\frac{\alpha_{x0}}{\rho}$$

so,

$$\alpha_x = \alpha_{x0} \cos\phi + \alpha_{y0} \sin\phi$$

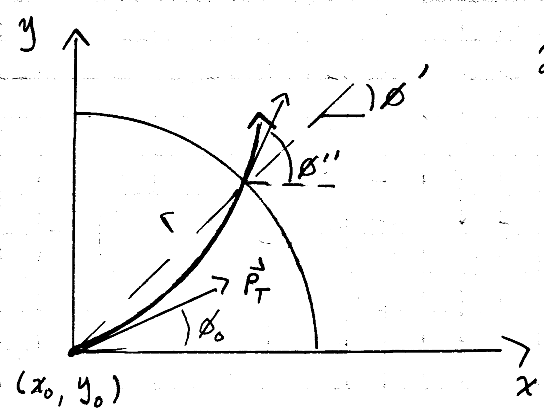
$$\alpha_y = -\alpha_{x0} \sin\phi + \alpha_{y0} \cos\phi$$

Re-write in Matrix notation

$$\Rightarrow \begin{pmatrix} \alpha_x \\ \alpha_y \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \alpha_{x0} \\ \alpha_{y0} \end{pmatrix} \qquad (1)$$

physically, $\phi(s)$ is the angle through which the momentum vector is rotating perpenduculer to $\vec{B}$ when the particle traverses a distance s.

$\vec{B} = B\hat{z}$



$\hat{\alpha}_T = \acute{\alpha} - \alpha$

Integrate again to find the position

$$\frac{d}{ds}\vec{x}(s) = \vec{\alpha}(s), \qquad \vec{x}(0) = \vec{x}_0 = (x_0, y_0, 0)$$

$$\vec{x}(s) = \int_0^s ds\, \vec{\alpha}(s)$$

$$= \begin{pmatrix} \rho\sin\phi & -\rho\cos\phi \\ \rho\cos\phi & +\rho\sin\phi \end{pmatrix} \begin{pmatrix} \alpha_{x0} \\ \alpha_{y0} \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$$

$$\vec{x}(0) = \begin{pmatrix} 0 & -\rho \\ \rho & 0 \end{pmatrix} \begin{pmatrix} \alpha_{x0} \\ \alpha_{y0} \end{pmatrix} + \begin{pmatrix} A \\ B \end{pmatrix}$$

$$-\alpha_{y0} + A = x_0$$

$$\alpha_{x0} + B = y_0$$

$$\Rightarrow \quad A = \rho\,\alpha_{y0} + x_0$$

$$B = -\rho\,\alpha_{x0} + y_0$$

$$x = \rho \sin\phi\, \alpha_{x0} - \rho\cos\phi\, \alpha_{y0} + \rho\,\alpha_{y0} + x_0$$

$$y = \rho\cos\phi\, \alpha_{y0} + \rho\sin\phi\,\alpha_{y0} - \rho\,\alpha_{x0} + y_0$$

$$\frac{x - x_0}{\rho} = \sin\phi\, \alpha_{x0} + (1-\cos\phi)\,\alpha_{y0}$$

$$\frac{y-y_0}{\rho} = \sin\phi\,\alpha_{y0} + (1-\cos\phi)(-\alpha_{x0})$$

$$\Rightarrow \quad \frac{\Delta\vec{x}}{\rho} = \sin\phi \begin{pmatrix} \alpha_{x0} \\ \alpha_{y0} \end{pmatrix} - (1-\cos\phi)\begin{pmatrix} -\alpha_{y0} \\ \alpha_{x0} \end{pmatrix} \qquad (2)$$

Now ellim*i*nate $\vec{s}$:

$$\frac{\Delta x}{\rho} = -(\alpha_y - \alpha_{y0}) \qquad \text{by (1)}$$

$$\frac{\Delta y}{\rho} = (\alpha_x - \alpha_{x0})$$

$$(\Delta x - \rho\,\alpha_{y0})^2 + (\Delta y + \rho\,\alpha_{x0})^2 = \rho\,(\alpha_y^2 + \alpha_x^2)$$

$$\equiv \rho_T^2$$

$$= \rho\,(\alpha_{x0}^2 + \alpha_{y0}^2)$$

we Know $|\hat{\alpha} \times \hat{\beta}| = |\hat{\alpha}||\hat{\beta}| \sin\theta = \sin\theta = \alpha_x^2 + \alpha_y^2 = \alpha_{x0}^2 + \alpha_{y0}^2$

$$P_T = p \sin\theta$$

$$\rho_T = \frac{P_T}{qB}$$

Invert (2):

$$\sin\phi = \frac{\left(\frac{\Delta x}{\rho} + \alpha_{y_0}\right)\alpha_{x0} + \left(\frac{\Delta y}{\rho} - \alpha_{x0}\right)\alpha_{y0}}{\left(\alpha_{x0}^2 + \alpha_{y0}^2\right)}$$

$$\cos\phi = \frac{\left(\frac{\Delta x}{\rho} + \alpha_{y_0}\right)\alpha_{x0} + \left(\frac{\Delta y}{\rho} - \alpha_{x0}\right)\alpha_{y0}}{\left(\alpha_{x0}^2 + \alpha_{y0}^2\right)}$$

We now have 3 of the fundamental track parameters:

$$\frac{q}{p}, \; \lambda, \; \phi$$

If (x,y) is known, we can find $\phi(x,y)$. Then from $\phi = \frac{s}{\rho}$ we can find z,

We have obvious cylindrical symmetry: switch to cylindrical coords:

For simplicity $x_0 = y_0 = 0$

$$x = r\cos\phi', \quad y = r\sin\phi'$$

$$\left(r\cos\phi' - \rho\alpha_{y0}\right)^2 + \left(r\sin\phi' + \rho\alpha_{x0}\right)^2$$

$$= r^2\cos^2\phi' - 2r\rho\alpha_{y0}\cos\phi' + \rho^2\alpha_{y0}^2$$

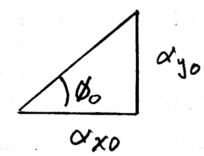
$$+ r^2\sin^2\phi' + 2r\rho\alpha_{x0}\sin\phi' + \rho^2\alpha_{x0}^2$$

$$= r^2 + \underbrace{\rho\left(\alpha_{x0}^2 + \alpha_{y0}^2\right)}_{\rho_T^2} + 2r\rho\left(\alpha_{x0}\sin\phi - \alpha_{y_0}\cos\phi'\right)$$

$$\rho_T^2 = r^2 + \rho_T^2 + 2\rho r\left(\alpha_{x0}\sin\phi' - \alpha_{y0}\cos\phi'\right)$$

$$-\frac{r}{2\rho} = \alpha_{x0}\sin\phi' - \alpha_{y0}\cos\phi'$$

Use $\tan\phi_0 = \frac{P_{0y}}{P_{0x}} = \frac{\alpha_{y0}}{\alpha_{x0}}$



$$\sin\phi_0 = \frac{\alpha_{y0}}{\sqrt{\alpha_{x0}^2 + \alpha_{y0}^2}} \qquad \cos\phi_0 = \frac{\alpha_{x0}}{\sqrt{\alpha_{x0}^2 + \alpha_{y0}^2}}$$

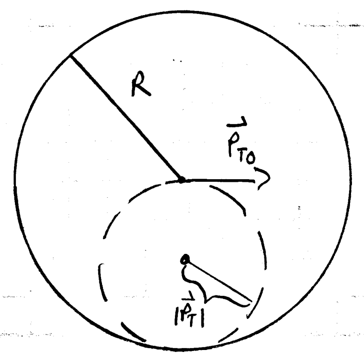
$$\rho^2(\alpha_{x0}^2 + \alpha_{y0}^2) = \rho_T^2$$

$\Rightarrow$ $$-\frac{r}{2\rho_T} = \cos\phi_0\sin\phi' - \sin\phi_0\cos\phi'$$

$$\boxed{\sin(\phi' - \phi_0) = \frac{r}{2\rho_T}}$$

If $\rho_T < \frac{R}{2}$ then track will never reach R

"curlers"



using $\rho_T = \frac{P_T}{qB}$ $\Rightarrow$ $P_T < \frac{RqB}{2} = 0.3\frac{R}{2}B$

$[P_T] = \text{GeV}$
$[R] = \text{m}$
$[B] = \text{T}$

CMS: Tracker Barrel radius $\gtrsim$ 1m

$$\frac{R}{2}qB = \frac{1}{2}(1\text{m})(e)(4\text{T}) = 0.6\ \text{GeV}$$

To find the exit vector (i.e. ϕ'').

$$\alpha_y = \alpha_{yo} - \frac{x}{\rho}$$

$$\alpha_x = \alpha_{xo} + \frac{y}{\rho}$$

$$\boxed{\tan \phi'' = \frac{\alpha_y}{\alpha_x} = \frac{\sin \phi_0 - \frac{r}{\rho_T} \cdot \cos \phi'}{\cos \phi_0 + \frac{r}{\rho_T} \sin \phi'}}$$

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